CSI Robust Joint User Selection and Precoder Design in MIMO Downlink Systems

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***Abstract*—In multiuser multiple-input multiple-output (MU-MIMO) systems, the selection of a subset of users to achieve the maximum sum rate is critical when resources are limited. In addition, designing suitable precoder and decoder matrices at the base station (BS) and at the user side requires unattainable perfect channel state information. In this letter, the downlink of a cellular system with multiple antennas at user equipments (UEs) is considered. A framework is derived for joint user selection and precoder/decoder design in the presence of imperfect channel estimation with neither instantaneous nor statistical error matrices. A robust method is proposed that incorporates a bounded worst-case channel in the design of precoders, decoders, and user selection. The optimization problem is solved iteratively by decomposition into multiple convex sub-problems which are solved successively. Finally, it is illustrated that for the case of a set of predesigned precoders, the interference pattern remains fixed for each user, implying that the problem of user selection could be transformed to that of beam selection.**

***Index Terms*—MIMO, Precoding, Channel State Information.**

I. Introduction

MULTIPLE antenna techniques aim to increase the sum-rate of wireless systems via spatial multiplexing. In particular, transmit and receive beamforming strategies for the cellular downlink are required. Emerging applications, e.g., Internet of Things (IoT) and massive machine-type communication (MMTC), feature *overloaded* networks, where user selection is critical since the BS can only serve a limited number of UEs simultaneously.

There has been significant attention in the literature on joint user selection and beamformer design in MIMO systems [1]. User selection for the SISO system downlink was investigated in [2]. In [3], user selection for OFDM systems was investigated, taking into account channel state information as selection criteria. Learning-based single-antenna user selection has been proposed in [4] using a proportional fairness metric. In [5], heuristic user selection was combined with an uplink-downlink duality method. In [6], a method was proposed for joint user selection and ZF-based precoder design with lower complexity than dirty paper coding. Joint user selection and beamforming for single-antenna users with perfect channel state information (CSI) was considered in [7] and for imperfect CSI in [8], [9]. For multi-antenna users, greedy user selection was proposed in [10] based on a selection metric derived from principal angles between subspaces. Block-diagonal (BD) precoding has been used for multi-antenna receivers in [11]. Greedy user selection along with fully digital BD precoders was deployed in [12] once a given set of users is selected. Rather than perform user selection with an exhaustive or greedy incremental search guided by a sum-rate metric, our objective here is to achieve a jointly optimized and flexible user selection strategy where group size is variable as a by-product of the optimization.

On the other hand, to nullify the interference in MU-MIMO systems, accurate knowledge of channel matrices at the BS is a requirement. However, in practical systems, access to perfect channel information at the BS is infeasible due to pilot contamination, quantization, and feedback limitations.

The robust design of MU-MIMO systems incorporates imperfect CSI assuming that an actual channel is within a neighbourhood of a nominal channel [13]. Robust strategies have been incorporated into optimization of MU-MIMO systems for non-orthogonal multiple access (NOMA) [14], cognitive radio [14] and regularized zero forcing precoding [15]. However, robust design of MU-MIMO systems for joint user selection and precoding has yet to be proposed.

This motivates the investigation in this letter into the impact of imperfect channel information on the user selection procedure. A well-known upper bound for the number of users in multi-antenna systems to fully nullify mutual interference among users is given by $\frac{M}{N}$, where M and N represent numbers of antennas at the transmitter and receiver, respectively. Our key contribution is the formulation of joint user selection and precoder design. Rather than availability of channel statistics, an elliptical uncertainty region characterizes the channel estimation error, e.g., bounded error matrix norm. An iterative solution is derived for over the worst-case channel. As a by-product, it is shown that in cases where the set of precoders/decoders are fixed, the mean squared error (MSE) experienced by each user is solely a function of the precoder assigned to that user regardless of selection of other users. This transforms the user selection problem into one of beam selection and makes the proposed method scalable for large-scale MIMO systems, potentially balancing performance and computational efficiency.

The rest of this letter is organized as follows: the system model and problem formulation for imperfect channel estimation are described in Sections II and III, respectively.

Received 1 March 2025; accepted 29 March 2025. Date of publication 2 April 2025; date of current version 11 June 2025. This work was supported in part by the Huawei Technologies Canada, and in part by the Natural Sciences and Engineering Research Council of Canada Discovery under Grant RGPIN-2019-06237. The associate editor coordinating the review of this article and approving it for publication was X. Li. *(Corresponding author: Hossein Vaezy.)*

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Digital Object Identifier 10.1109/LWC.2025.3557052

Equivalence of user scheduling and beamforming assignment is presented in Section IV and simulation results in Section V.

II. System Model

The downlink of a MU-MIMO network is considered, where a BS is equipped with M antennas that transmit to K UE receivers denoted by the set $\mathcal{K}=\{1,2,\ldots,K\}$. Each UE has N antennas to support d data symbols, resulting in a total of Kd data symbols transmitted by the BS. In order to ensure feasible precoding and decoding, the number of transmitted symbols is restricted by $Kd \leq M$ and $d \leq N$ for BS and UEs, respectively.

Let $\mathbf{H}_k \in \mathbb{C}^{N\times M}$ denote the complex channel between the BS and UE k. The transmitted signal intended for UE k is given by $\mathbf{V}_k \boldsymbol{s}_k$ where $\mathbf{V}_k$ denotes the linear precoder matrix, and $\boldsymbol{s}_k \in \mathbb{C}^{d\times 1}$ is UE k's normalized data symbol, i.e., $\mathrm{E}\{\boldsymbol{s}_k \boldsymbol{s}_k^H\} = 1$. The received signal vector at UE k is given by

$$\boldsymbol{y}_k = \sum_{\ell=1}^{K} x_\ell \mathbf{H}_k \mathbf{V}_\ell \boldsymbol{s}_\ell + \boldsymbol{n}_k,\ \forall k \in \mathcal{K}, \tag{1}$$

where, $\boldsymbol{n}_k \in \mathbb{C}^{N\times 1}$ denotes the noise vector at UE k. The term x_l represents the user indicator function that takes the value 1 for a user selected for transmission and 0 otherwise. At the receiver side, UE k first processes the received signals using decoder matrix, $\mathbf{W}_k$. Therefore, the estimated symbol

$$\hat{\boldsymbol{s}}_k = x_k \mathbf{W}_k^H \mathbf{H}_k \mathbf{V}_k \boldsymbol{s}_k + \sum_{\ell\neq k}^{K} x_\ell \mathbf{W}_k^H \mathbf{H}_k \mathbf{V}_\ell \boldsymbol{s}_\ell + \mathbf{W}_k^H \boldsymbol{n}_k. \tag{2}$$

The first term on the right-hand side of (2) is the desired signal for UE k, the second term is the aggregate interference, and the last term denotes the filtered noise.

By assuming Gaussian-distributed transmitted symbols and a circularly symmetric complex Gaussian noise random vector with zero mean and covariance matrix $\mathrm{E}\{\boldsymbol{n}_k \boldsymbol{n}_k^H\} = \sigma^2 \mathbf{I}_N$, the achievable rate for UE k is shown to be [15]:

$$R_k = \log_2 \det\left(\mathbf{I}_d + x_k^2 \mathbf{V}_k^H \mathbf{H}_k^H \mathbf{W}_k \mathbf{C}_k^{-1} \mathbf{W}_k^H \mathbf{H}_k \mathbf{V}_k\right),\ \forall k \in \mathcal{K},$$

where UE k's interference covariance matrix is given by:

$$\mathbf{C}_k = \mathbf{W}_k^H \left(\sigma^2 \mathbf{I} + \sum_{\ell\neq k}^{K} x_\ell^2 \mathbf{H}_k \mathbf{V}_\ell \mathbf{V}_\ell^H \mathbf{H}_k^H\right) \mathbf{W}_k,\ \forall k \in \mathcal{K}. \tag{3}$$

III. Bounded Channel Uncertainty

Typically, the channel is assumed to be perfectly known at the BS and UEs. However, in practice, CSI is noisy due to feedback error, finite-resolution ADC, and quantization error. To address imperfect CSI, most of the literature employs channel characterizations based on first- and second-order statistics. Here, neither instantaneous nor channel estimation error statistics are assumed. Instead, the error matrix has an elliptically bounded uncertainty region. The worst-case channel realization is used for user subset selection and for precoder/decoder design. Channel estimation is modeled as

$$\mathbf{H}_k = \bar{\mathbf{H}}_k + \mathbf{E}_k, \tag{4}$$

where $\bar{\mathbf{H}}_k \in \mathbb{C}^{N\times M}$ and $\mathbf{E}_k \in \mathbb{C}^{N\times M}$ represent the kth UE's estimated channel and channel estimation error matrices at the BS, respectively. Using (4), the received signal at the kth UE

$$\boldsymbol{y}_k = \sum_{\ell=1}^{K} x_k (\bar{\mathbf{H}}_k + \mathbf{E}_k) \mathbf{V}_\ell \boldsymbol{s}_\ell + \boldsymbol{n}_k. \tag{5}$$

Using (2), the symbols estimated by the kth UE are then

$$\hat{\boldsymbol{s}}_k = x_k \mathbf{W}_k^H (\bar{\mathbf{H}}_k + \mathbf{E}_k) \mathbf{V}_k \boldsymbol{s}_k + \sum_{\ell\neq k}^{K} x_\ell \mathbf{W}_k^H (\bar{\mathbf{H}}_k + \mathbf{E}_k) \mathbf{V}_\ell \boldsymbol{s}_\ell + \mathbf{W}_k^H \boldsymbol{n}_k, \tag{6}$$

and the achievable rate of UE k is given by

$$R_k = \log_2 \det\Big(\mathbf{I}_d + x_k^2 \mathbf{V}_k^H (\bar{\mathbf{H}}_k + \mathbf{E}_k)^H \mathbf{W}_k \mathbf{C}_k^{-1} \mathbf{W}_k^H (\bar{\mathbf{H}}_k + \mathbf{E}_k) \mathbf{V}_k\Big),\ \forall k \in \mathcal{K}. \tag{7}$$

A. Problem Formulation

Our target objective is to select a subset of UEs for simultaneous transmission and design precoders and decoders to maximize the worst-case total sum rate while being robust to channel uncertainty as described above. The optimization problem can therefore be cast as follows:

$$\max_{\{x_k,\mathbf{V}_k,\mathbf{W}_k\}_{k\in\mathcal{K}}}\ \min_{\{\mathbf{E}_k\}_{k\in\mathcal{K}}}\ \sum_{k=1}^{K} \alpha_k R_k \tag{8a}$$

$$\text{subject to}\quad \sum_{k=1}^{K} \mathrm{tr}(\mathbf{V}_k \mathbf{V}_k^H) \leq P_{max},\ \forall k \in \mathcal{K}, \tag{8b}$$

$$\mathrm{tr}\Big(\mathbf{E}_k^H \mathbf{A}_k \mathbf{E}_k\Big) \leq \beta_k,\ \forall k \in \mathcal{K}, \tag{8c}$$

$$x_k \in \{0,1\},\ \forall k \in \mathcal{K}, \tag{8d}$$

$$\sum_{k=1}^{K} x_k \leq K_{max}, \tag{8e}$$

where P_{max} is the total power constraint at the BS, and $\alpha_k \in [0,1]$ denotes user k's priority, where larger α indicates higher priority. Parameters $\mathbf{A}_k \succeq 0$ and $\beta_k \geq 0$ jointly denote the feasible ellipsoid region for $\mathbf{E}_k$ [16], while K_{max} represents an a priori known maximum number of served UEs. If K_{max} is chosen to be large, the constraint is inactive and there is no constraint on the number of served UEs. On the other hand, K_{max} can be used as an adjustment parameter to satisfy the constraint $K_{max} \leq \frac{M}{N}$ to ensure decoding.

Optimization problem (8), is a max-min problem along with integer variables that makes the problem non-convex. Therefore, finding the optimal point for this joint optimization problem is likely to be intractable. However, even with the set of users fixed, this problem is non-convex and cannot be solved directly by conventional convex optimization techniques.

Lemma 1: Suppose $\mathbf{S} \in \mathbb{C}^{d\times d}$ is a positive definite matrix. Maximization of $\log_2 |\mathbf{S}^{-1}|$ corresponds to the following:

$$\max_{\mathbf{S},\mathbf{Z}}\ \log_2 |\mathbf{Z} \ln 2| - \mathrm{tr}(\mathbf{S}\mathbf{Z}) + \frac{d}{\ln 2}, \tag{9}$$

where $\mathbf{Z} \in \mathbb{C}^{d\times d}$ is a positive definite matrix.

Proof: For a given $\mathbf{S}$, as a result of first-order optimality conditions for the convex problem (9), the optimal $\mathbf{Z}$, i.e., $\mathbf{Z}^\star$ can be found as follows:

$$\mathbf{Z}^\star = \frac{\mathbf{S}^{-1}}{\ln 2}. \tag{10}$$

By substituting (10) into (9), Lemma 1 is proved. ∎

Using Lemma 1 and the relation between mean-squared error and rate [17], optimization problem (8) is recast as:

$$\max_{\{x_k,\mathbf{V}_k,\mathbf{W}_k\}_k} \min_{\{\mathbf{E}_k\}_{k=1}^K} \max_{\{\mathbf{Z}_k\}_{k=1}^K} \sum_{i=1}^{K} \Big\{\alpha_i \log_2|\mathbf{Z}_i| - \alpha_i \mathrm{tr}(\mathbf{Z}_i\mathbf{MSE}_i) + \frac{\alpha_i d}{\ln 2}\Big\} \tag{11}$$

subject to (8b), (8c), (8d), (8e) where

$$\mathbf{MSE}_\ell = x_\ell(\mathbf{W}_\ell^H\mathbf{H}_\ell\mathbf{V}_\ell - \mathbf{I})x_\ell(\mathbf{W}_\ell^H\mathbf{H}_\ell\mathbf{V}_\ell - \mathbf{I})^H + \sum_{j\neq\ell}^{K} x_j^2\mathbf{W}_\ell^H\mathbf{H}_\ell\mathbf{V}_j\mathbf{V}_j^H\mathbf{H}_\ell^H\mathbf{W}_\ell + \sigma^2\mathbf{W}_\ell^H\mathbf{W}_\ell. \tag{12}$$

Due to the triple max-min-max, Problem (11) is complicated to solve. Replacing the inner min-max by a max-min, the objective function can be lower bounded as:

$$\max_{\{x_k,\mathbf{V}_k,\mathbf{W}_k,\mathbf{Z}_k\}_k} \min_{\{\mathbf{E}_k\}_{k=1}^K} \sum_{i=1}^{K}\Big\{\alpha_i\log_2|\mathbf{Z}_i| - \alpha_i\mathrm{tr}(\mathbf{Z}_i\mathbf{MSE}_i) + \frac{\alpha_i d}{\ln 2}\Big\}$$
$$\text{subject to } (8b), (8c), (8d), (8e). \tag{13}$$

To further simplify (13), we define $\mathbf{F}_i$ such that $\mathbf{F}_i\mathbf{F}_i^H \triangleq \mathbf{Z}_i$, and using trace property, $\mathrm{tr}(\mathbf{AB}) = \mathrm{vec}(\mathbf{A}^H)^H\mathrm{vec}(\mathbf{B})$, $\mathrm{tr}(\mathbf{Z}_i\mathbf{MSE}_i)$ can be expressed as a sum of $k-$vector norms:

$$\mathrm{tr}(\mathbf{Z}_i\mathbf{MSE}_i) = \|\mathrm{vec}\Big((x_i\mathbf{F}_i^H(\mathbf{W}_i^H\mathbf{H}_i\mathbf{V}_i - \mathbf{I}_d)\Big)\|^2 + \sum_{\ell\neq i}^{K} x_\ell\|\mathrm{vec}\Big(\mathbf{F}_i^H\mathbf{W}_i^H\mathbf{H}_i\mathbf{V}_\ell\Big)\|^2 + \sigma^2\|\mathrm{vec}(\mathbf{W}_i\mathbf{F}_i)\|^2. \tag{14}$$

Now, by substituting $\mathbf{H}_i = \bar{\mathbf{H}}_i + \mathbf{E}_i$, our objective is to separate the contribution of $\mathbf{E}_k$, $\forall k \in \mathcal{K}$. Using the identity $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{B})$, Eq. (14) becomes:

$$\mathrm{tr}(\mathbf{Z}_i\mathbf{MSE}_i) = \sum_{\ell=1}^{K}\|\mathbf{b}_{i\ell} + \mathbf{B}_{i\ell}\mathrm{vec}(\mathbf{E}_i)\|^2, \tag{15}$$

where $\mathbf{b}_{i\ell} \in \mathbb{C}^{n_{i\ell}\times 1}$ and $\mathbf{B}_{i\ell} \in \mathbb{C}^{n_{i\ell}\times MN}$ are defined as:

$$\begin{aligned}
\mathbf{b}_{i\ell} &\triangleq \mathrm{vec}(x_\ell\mathbf{F}_i^H\mathbf{W}_i^H\bar{\mathbf{H}}_i\mathbf{V}_\ell) \\
\mathbf{B}_{i\ell} &\triangleq x_\ell\mathbf{V}_\ell^T \otimes \mathbf{F}_i^H\mathbf{W}_i^H,\ \forall\ell\neq i \\
\mathbf{b}_{ii} &\triangleq \begin{bmatrix}\mathrm{vec}\big(x_i\mathbf{F}_i^H(\mathbf{W}_i^H\bar{\mathbf{H}}_i\mathbf{V}_i - \mathbf{I}_d)\big)\\ \sigma\mathrm{vec}\big(\mathbf{F}_i^H\mathbf{W}_i^H\big)\end{bmatrix} \\
\mathbf{B}_{ii} &\triangleq \begin{bmatrix}x_i\mathbf{V}_i^T \otimes \mathbf{F}_i^H\mathbf{W}_i^H\\ \mathbf{0}_{Nd\times MN}\end{bmatrix} \\
n_{i\ell} &\triangleq \begin{cases} Nd + d^2, & i = \ell\\ d^2, & i\neq\ell.\end{cases}
\end{aligned} \tag{16}$$

Due to the inner minimization problem, the latter optimization problem is still hard to solve. By introducing slack variables $\mu_{i\ell}$ we instead formulate the alternative optimization problem:

$$\max_{\{x_k,\mathbf{V}_k,\mathbf{W}_k,\mathbf{F}_k,\mu_{k,j}\}_{k,j}} \sum_{i=1}^{K}\Big\{2\alpha_i\log_2|\mathbf{F}_i| - \alpha_i\sum_{\ell=1}^{K}\mu_{i\ell} + \frac{\alpha_i d}{\ln 2}\Big\} \tag{17}$$

subject to (8b), (8c), (8d), (8e),

$$\|\mathbf{b}_{i\ell} + \mathbf{B}_{i\ell}\mathrm{vec}(\mathbf{E}_i)\|^2 \leq \mu_{i\ell},\ \forall i,\ell\in\mathcal{K}. \tag{17a}$$

Using Schur complement, condition (17a) can be cast in linear matrix inequality form as follows:

$$\begin{bmatrix}\mu_{i\ell} & \mathbf{b}_{i\ell}^H + \mathrm{vec}(\bar{\mathbf{E}}_i)^H\bar{\mathbf{A}}_i^H\mathbf{B}_{i\ell}^H\\ \mathbf{b}_{i\ell} + \mathbf{B}_{i\ell}\bar{\mathbf{A}}_i\mathrm{vec}(\bar{\mathbf{E}}_i) & \mathbf{I}_{n_{i\ell}}\end{bmatrix} \succeq 0, \tag{18}$$

where, $\bar{\mathbf{E}}_i \triangleq \mathbf{A}_i\mathbf{E}_i$ and $\bar{\mathbf{A}}_i \triangleq \mathbf{I}\otimes\mathbf{A}_i^{-1}$.

To express the two norm-bounded constraints in (8e) and (18) as a single constraint and further simplify the optimization problem, the following lemma can be used [16].

Lemma 2: Given matrices $\mathbf{P}, \mathbf{Q}, \mathbf{A}$ with $\mathbf{A} = \mathbf{A}^H$

$$\mathbf{A} \succeq \mathbf{P}^H\mathbf{X}\mathbf{Q} + \mathbf{Q}^H\mathbf{X}^H\mathbf{P},\ \forall\mathbf{X}: \|\mathbf{X}\|\leq\epsilon \tag{19}$$

if and only if there exists a $\lambda \geq 0$ such that

$$\begin{bmatrix}\mathbf{A} - \lambda\mathbf{Q}^H\mathbf{Q} & -\epsilon\mathbf{P}^H\\ -\epsilon\mathbf{P} & \lambda\mathbf{I}\end{bmatrix} \succeq 0. \tag{20}$$

Proof: See [16]. ∎

The matrices in Lemma 2 can be chosen as follows:

$$\mathbf{A} = \begin{bmatrix}\mu_{i\ell} & \mathbf{b}_{i\ell}^H\\ \mathbf{b}_{i\ell} & \mathbf{I}_{n_{i\ell}}\end{bmatrix},\ \mathbf{P} = \begin{bmatrix}\mathbf{0}_{Mn\times 1} & \bar{\mathbf{A}}_i^H\mathbf{B}_{i\ell}^H\end{bmatrix}$$
$$\mathbf{Q} = \begin{bmatrix}-1 & \mathbf{0}_{1\times n_{i\ell}}\end{bmatrix},\ \mathbf{X} = \mathrm{vec}\big(\bar{\mathbf{E}}_{i\ell}\big). \tag{21}$$

Finally, the optimization problem (17) can be rewritten as

$$\max_{\{x_k,\mathbf{V}_k,\mathbf{W}_k,\mathbf{F}_k,\mu_{k,j},\lambda_{k,j}\}_{k,j}} \sum_{i=1}^{K}\Big\{2\alpha_i\log_2|\mathbf{F}_i| - \alpha_i\sum_{\ell=1}^{K}\mu_{i\ell} + \frac{\alpha_i d}{\ln 2}\Big\} \tag{22}$$

subject to (8d), (8e)

$$\begin{bmatrix}P_{max} & \mathrm{vec}(\mathbf{V})^H\\ \mathrm{vec}(\mathbf{V}) & \mathbf{I}_{Md}\end{bmatrix} \succeq 0, \tag{22a}$$

$$\begin{bmatrix}\mu_{i\ell} - \lambda_{i\ell} & \mathbf{b}_{i\ell}^H & \mathbf{0}_{1\times MN}\\ \mathbf{b}_{i\ell} & \mathbf{I}_{n_{i\ell}} & -\beta_i\mathbf{B}_{i\ell}\bar{\mathbf{A}}_i\\ \mathbf{0}_{MN\times 1} & -\beta_i\bar{\mathbf{A}}_i^H\mathbf{B}_{i\ell}^H & \lambda_{i\ell}\mathbf{I}_{MN}\end{bmatrix} \succeq 0,\ \forall i,\ell\in\mathcal{K}, \tag{22b}$$

where, $\mathbf{V} = [\mathbf{V}_1, \ldots, \mathbf{V}_K] \in \mathcal{C}^{M\times Kd}$.

While optimization problem (22) is jointly non-convex over variables $\{x_k, \mathbf{V}_k, \mathbf{W}_k, \mathbf{F}_k\}_k$, it is convex for each variable individually. This suggests an iterative method in which three of the four parameters are successively fixed and the remaining parameter is optimized. For given set $\{x_k, \mathbf{W}_k, \mathbf{F}_k\}_k$, an SDP problem is solved to obtain precoder matrices. For given $\{x_k, \mathbf{V}_k, \mathbf{F}_k\}_k$, decoder matrices are then computed by SDP. For given $\{x_k, \mathbf{V}_k, \mathbf{W}_k\}_k$, $\{\mathbf{F}_k\}_k$ are obtained by maximization of $\log|\cdot|$ using the MAX-DET algorithm [18]. For given set $\{\mathbf{V}_k, \mathbf{W}_k, \mathbf{F}_k\}_k$, a subset of users can in principle be selected by relaxing $\{x_k\}$ to real-valued, solving an SDP program, and then discretizing to nearest 0 or 1. This approach inherently introduces a level of error during the user selection

Algorithm 1: Proposed Joint UE Selection and Hybrid Precoder/Decoder Design

1 Initialize $\{\mathbf{V}_k,\ \mathbf{W}_k,\ \mathbf{F}_k\}_{k\in\mathcal{K}}$ to satisfy the power and $x_k = 1, \forall k$.
2 $j = 1$
3 **while** $\mid f^{(j)} - f^{(j-1)} \mid > \epsilon_{stop}$ **do**
4 Given $\{x_k,\ \mathbf{V}_k,\ \mathbf{W}_k\}_{k\in\mathcal{K}}$, compute $\{\mathbf{F}_k\}_{k\in\mathcal{K}}$ by solving (22) using MAX-DET algorithm.
5 Given $\{x_k,\ \mathbf{V}_k,\ \mathbf{F}_k\}_{k\in\mathcal{K}}$, compute $\{\mathbf{W}_k\}_{k\in\mathcal{K}}$ by solving (22).
6 Given $\{x_k,\ \mathbf{W}_k,\ \mathbf{F}_k\}_{k\in\mathcal{K}}$, compute $\{\mathbf{V}_k\}_{k\in\mathcal{K}}$ by solving (22).
7 $j \leftarrow j + 1$
8 **end**
9 Given $\{\mathbf{V}_k,\ \mathbf{W}_k,\ \mathbf{F}_k\}_{k\in\mathcal{K}}$, compute $\{x_k\}_{k\in\mathcal{K}}$ by solving (24).

phase. Rather than solving (22), an alternative approach is used that incorporates user selection directly into the orginal slack variable optimization problem. The impact of user selection parameters $\{x_k\}$ on problem (17) can be expressed as follows:

$$\begin{aligned} \min_{\{x_k,\mu_{k,j}\}_{k,j}} \quad & \sum_{i=1}^{K}\sum_{\ell=1}^{K} \alpha_i . x_\ell . \mu_{i\ell} + (1-x_i)\eta_i && (23)\\ \text{subject to} \quad & (8d),\ (8e), \\ & \|\mathbf{b}_{i\ell} + \mathbf{B}_{i\ell}\text{vec}(\mathbf{E}_i)\|^2 \le \mu_{i\ell},\ \forall i,\ell \in \mathcal{K} && (23a) \end{aligned}$$

where $\eta_i = \|\sigma\text{vec}(\mathbf{F}_i^H\mathbf{W}_i^H)\|^2$. It is obvious that $\mu_{i,\ell}$ is optimized in lines 4, 5, 6 of Algorithm 1. Hence, user selection optimization is equivalently described as:

$$\begin{aligned} \min_{\{x_k\}_k} \quad & \sum_{\ell=1}^{K} x_\ell \left(\sum_{i=1}^{K} \alpha_i . \mu_{i\ell} - \eta_\ell \right) && (24)\\ \text{subject to} \quad & x_k \in \{0,1\},\ \forall k = 1,\ldots,K, \\ & \sum_{i=1}^{K} x_i \le K_{max}. && (24a) \end{aligned}$$

Finally, it can be seen that for $\sum_{i=1}^{K}\alpha_i.\mu_{i\ell} - \eta_\ell \ge 0$, UE ℓ will not be scheduled, i.e., $x_\ell = 0$ and for $\sum_{i=1}^{K}\alpha_i.\mu_{i\ell} - \eta_\ell < 0$ UE *m* will be scheduled if *m* is among the minimum K_{max} of the set $\{\sum_{i=1}^{K}\alpha_i.\mu_{i\ell} - \eta_\ell\}_{\ell=1}^{K}$.

The above steps of the proposed method are summarized in Algorithm 1, where $f^{(j)}$, the objective function (22), represents the sum rate lower bound at the *j*th iteration.

Remark 1: For given $\{x_k, \mathbf{V}_k, \mathbf{W}_k\}_k$, the MAX-DET problem in (22) is separable with respect to $\mathbf{F}_k, \forall k \in \mathcal{K}$. Hence, the problem can be decomposed into K individual sub-problems distributing complexity among users, thereby lowering overall complexity compared to the joint solution.

Remark 2: The computational complexity of the proposed framework is dominated by the $\log\det(\cdot)$ minimization problem which is known to be $O(M^{6.5})$ [18].

IV. On Equivalence of User Scheduling and Beamformer Assignment

In certain scenarios where the precoder, decoder and matrices $\{\mathbf{F}_k\}_k$ have been previously designed and remain fixed, it can be observed from Equation (12) that, $\mathbf{MSE}_i$ is a function of $\mathbf{V}_i$ and $\mathbf{W}_i$. This relation holds regardless of assignments of the other precoders/decoders to remaining UEs. To illustrate this, consider a scenario where the BS is serving UEs using a given set of N_z non-zero precoders, denoted as

$$\bar{\mathcal{V}} = \left\{ \mathbf{V}_n \in \mathbb{C}^{M\times d} | n = 1,\ldots,N_z,\ \|\mathbf{V}_n\|^2 \ne 0 \right\}. \quad (25)$$

Consider UE *m* that is either scheduled by the BS for transmission using one of the non-zero precoders or not scheduled. If scheduled, $\mathbf{MSE}_m$ is a superposition of desired signal, interference and noise power. If not scheduled, $\mathbf{MSE}_m$ consists primarily of interference and noise power. More specifically,

$$\mathbf{MSE}_m = \begin{cases} \eta_m & : x_m = 0 \\ \eta_m - 2\Re\{\mathbf{W}_m^H\mathbf{H}_m\mathbf{V}_m\} & : x_m = 1 \end{cases} \quad (26)$$

where $\eta_m = \mathbf{W}_m^H\mathbf{H}_m\sum_{\ell=1}^{N_z}(\mathbf{V}_\ell\mathbf{V}_\ell^H)\mathbf{H}_m^H\mathbf{W}_m + \sigma^2\mathbf{W}_m^H\mathbf{W}_m + \mathbf{I}_d$ is fixed for each UE with a specific set of precoders.

Remark 3: When UE *m* is scheduled with the *n*th non-zero precoder, $\mathbf{MSE}_m$ remains unaffected by other UEs scheduled on the remaining $(Nz - 1)$ precoders. Conversely, if UE *m* is not scheduled, $\mathbf{MSE}_m$ is fixed, independent of UE-precoder assignments. This implies that the user scheduling problem can be equated to assigning a set of fixed non-zero precoders to UEs, i.e., finding the best set of N_z UEs to be served at the BS by a set of fixed precoders. Also, complexity can be reduced by decreasing N_z. A practical use case of the above scenario is in mmWave networks where predefined analog precoders are deployed due to phase shifter constraints [19]. In dense networks, where the number of users exceeds the number of available beams, selecting the best set of users becomes equivalent that of optimal precoder assignment.

V. Simulation Results

Simulations are used to evaluate the proposed method in MU-MIMO systems. Without loss of generality, the estimated channel realizations between BS and UEs are generated to be Rayleigh distributed. The number of users, transmit antennas, and receive antennas are represented by *K*, *M*, and *N*, respectively, and $\epsilon_{stop} = 10^{-3}$ is used as the stopping criterion in Algorithm 1. Fig. 1 presents the worst-case sum rate for various combinations of transmit powers, P_{max}, and uncertainty region size parameters, β, with $M = 8$, $N = 2$, $d = 1$, $K = 10$, and $K_{max} = 8$. The performance metric is the worst-case sum rate over the uncertainty region, i.e., equivalent to objective function (22). The plotted points are averages over solutions to Problem (22) using 30 independent channel realizations. As seen in Fig. 1, decreasing transmit power results in sum-rate performance degradation. As expected, for $\beta = 0$ (and extended horizontal line) which corresponds to perfect channel knowledge, worst-case sum rate is equivalent to actual sum rate. In addition, in the low transmit power regime, performance only degrades slightly as β varies.

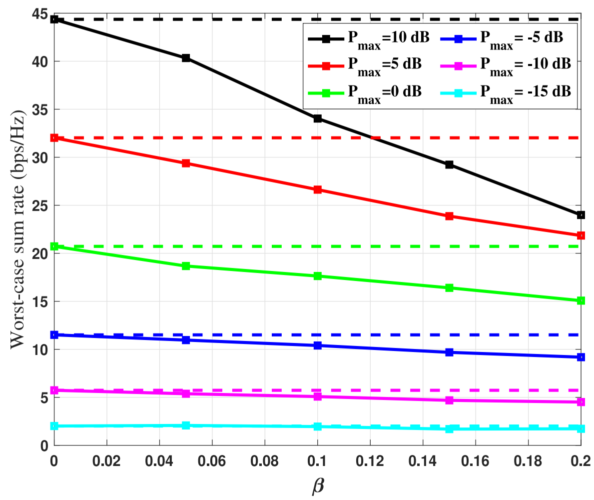

Fig. 1. Worst-case sum rate for $M = 8$, $N = 2$, $K = 10$, $K_{max} = 8$ with various β and P_{max}.

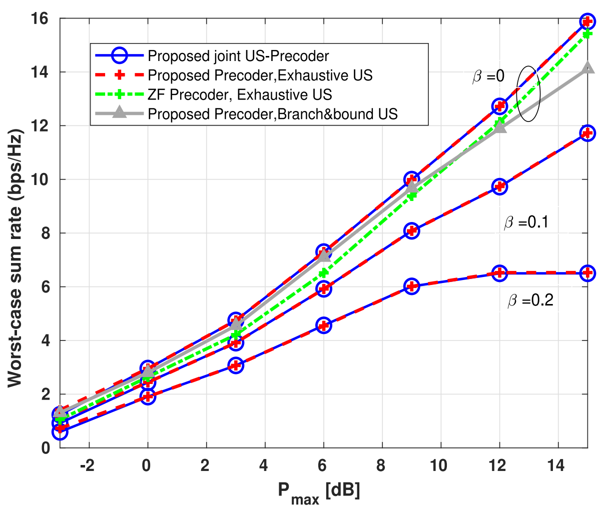

Fig. 2. Worst-case sum rate for $M = 8$, $N = 1$, $K = 10$, $K_{max} = 8$ with various β and P_{max}.

Fig. 2 compares the proposed method with exhaustive search-based user selection (US), Branch and Bound [20] US, and ZF precoding as benchmarks. For $K = 10$ and $M = 8$, precoders are designed for all possible user sets, resulting in $\binom{10}{8} = 45$ combinations. The set that achieves the maximum sum rate is selected as the optimal set of users. The proposed joint US and precoder design (blue circles) closely follows the performance of exhaustive user selection, demonstrating near-optimal performance with reduced complexity. Branch and Bound US also provides a competitive sum rate at significantly reduced computational overhead. As also observed, the proposed methods have superior performance compared to that of ZF, particularly at lower power levels, highlighting the benefits of the optimized precoder design.

VI. Conclusion

A method for jointly optimizing user selection, group size, precoders and decoders in the presence of bounded channel uncertainty is proposed based on worst-case sum rate. Subsets of parameters are determined in an alternating fashion by decomposing the joint problem into multiple sub-problems. Graceful degradation in performance is observed as the channel error uncertainty increases. Finally, it is observed that for a set of predesigned precoders, the interference pattern remains fixed for each user, which implies that the user selection problem could be reformulated as a beam selection problem.

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