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RESEARCH ARTICLE

Reduced Complexity Rate-Splitting Multiple Access Beamforming for Generalized Objectives

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ABSTRACT Rate splitting multiple access (RSMA) enables interference management trade-offs between space-division multiple access (SDMA) and non-orthogonal multiple access (NOMA) to serve multiple users in the multiple-input multiple-output (MIMO) broadcast channel. The design of RSMA beamforming, or precoding, to maximize typical rate performance is a non-convex optimization problem. In this paper, a parameterization of optimal beamforming directions for different performance maximization problems subject to a total power constraint is obtained for RSMA for systems with full column rank channel matrices. The performance evaluation function may be an arbitrary increasing function of split user rates. The proposed solution combines maximizing each stream's signal-to-noise-ratio (SNR) and reducing interference on other streams for different objectives. Using the derived beamforming directions, the design is completed by solving a power allocation problem. Simulation results reveal that the proposed approach is able to provide attractive performance/complexity trade-offs compared to existing schemes.

INDEX TERMS Rate-splitting multiple access, broadcast channel, multiple-input multiple-output communications, non-orthogonal multiple access, space-division multiple access.

I. INTRODUCTION

Fifth and sixth generation (5G/6G) wireless communications systems aim to provide services with ultra-reliability, low-latency, and high data rate, but face severe challenges to serve multiple users and manage interference. Multi-user multiple-input multiple-output (MU-MIMO) technology is essential to provide high data rate [1]. Rate-splitting multiple access (RSMA) is a recently proposed multiple access (MA) scheme that efficiently uses spatial degrees of freedom provided by MIMO, i.e., space-division multiple access (SDMA), and at the same time, takes advantage of the benefits of non-orthogonal multiple access (NOMA) to manage interference for a broad range of network loads and user deployments.

A. RELATED WORKS

To exploit the large spatial dimensionality offered by the MU-MIMO broadcast channel (BC), non-linear dirty paper coding (DPC) [2] is proposed for precoding that has high complexity. This motivates SDMA with linear precoding at the transmitter that treats multi-user interference in the MIMO BC as noise. The rate performance of SDMA approaches that of DPC for under-loaded networks, i.e., those where the number of base station (BS) antennas is greater than the number of users, as well as for the case that users' channel vectors are close to orthogonal and with similar strengths. Rate performance of SDMA, however, degrades dramatically for overloaded networks.

An alternative MA technique, NOMA, superimposes user signals at the transmitter and applies successive interference cancellation (SIC) at the receiver. The capacity region of NOMA is known to achieve that of the single-input single-output (SISO) BC [3]. There have been attempts to generalize NOMA to multi-antenna networks by combining SDMA and NOMA either with or without user grouping [4], [5], [6], [7], [8], [9]. However, for MIMO and multiple-input single-output (MISO) due to transmitter precoding, users' channels are not degraded, and as a result, precoding and SIC decoding

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order must be jointly optimized, which results in high NOMA complexity for multi-antenna networks. In overloaded networks with aligned channel vectors of highly differing gains, NOMA performs well [1]. In summary, SDMA and NOMA performances depend on network load and user deployment and cannot be efficiently combined.

The principle in RSMA is to split and combine users' messages into streams at the transmitter and apply partial interference decoding and partial treatment of interference as noise at the receivers. In this way, RSMA provides more flexible interference management that overcomes limitations of SDMA and NOMA [1], [10]. In [10], RSMA is initially proposed for 1-layer rate splitting (RS) with one common stream and then extended to the general case with at most $2^K - 1$ streams, where K is the number of users. Other low complexity schemes such as 2-layer RS and hierarchical streams are also introduced in [10], [11], and [12] and references in [1, Section II-B]. The precoding design and analysis presented in this paper is applicable to general and low complexity RSMA schemes with linear precoding.

Beamforming optimization and power allocation are crucial for RSMA to achieve its benefits in wireless communication networks [1]. However, generalizing SDMA to RSMA involves multicasting, and the beamforming design problem for multicasting is NP-hard [13]. Therefore, resource allocation optimization problems in RSMA are intractable and non-convex due to coupled user rate expressions. RSMA precoding designs are considered for perfectly-known channel state information at the transmitter (CSIT) [10], [12], [14], [15], [16], [17], [18], [19], [20] and partial or imperfect CSIT [11], [21], [22], [23], [24], [25], [26] to optimize performance, where the performance evaluation function includes weighted sum rate (WSR) [10], [14], [15], [16], [19], [20], [21], [22], minimum rate of users [17], [18], [22], [24], energy efficiency [23], [25], and sum-power consumption [26]. The algorithms used to approximate the solution to beamforming design optimization problems are weighted minimum mean square error (WMMSE)-based algorithms [10], [16], [21], [24] and successive convex approximation (SCA)-based algorithms [14], [22], [23], which are proven to converge to the well-known Karush-Kuhn-Tucker (KKT) points. Beamforming design is investigated for general RSMA [10], [20], [22] and its low complexity versions including 1-layer RS [14], [15], [16], [21], [24], [25], [26], 2-layer RS [10], [11], and RS based on hierarchical user grouping [12], [18]. Global optimality of 1-layer RS beamforming design for weighted sum rate maximization problem is obtained in [27] by the branch and bound (BB) method.

Although RSMA beamforming design has been investigated in the literature, optimal beamforming solutions have yet to be proposed unless restricted to certain performance evaluation functions or to specific RSMA special cases. For example, in [22], the precoding design problem is formulated as one of performance maximization subject to a total power constraint, where the performance is evaluated by weighted sum rate and max-min rate of users. The concave-convex procedure (CCCP) [28] is used to approximate the solution iteratively. Although the algorithm is proven to converge to a stationary point, the results are restricted to specific performance evaluation functions. In addition, the iterative precoding design algorithms do not provide intuition on how streams affect one other, and an explicit expression for beamforming vector solution is required.

B. MOTIVATION AND CONTRIBUTIONS

To reduce precoding complexity and provide a design method inspired by optimal beamforming, a parameterization for optimal beamforming directions of the performance maximization problem subject to a total power constraint is obtained for systems with full column rank channel matrices, where the objective function is of a general form that includes the following well-known performance metrics (system utility functions): *i)* weighted arithmetic mean (WAM) *ii)* weighted geometric mean (WGM), *iii)* weighted harmonic mean (WHM), and *iv)* weighted max-min (WMM) of users' rates [29], each of which provides a different trade-off between maximizing aggregate performance and fairness.

An explicit expression for beamforming directions is obtained from a set of parameters that specify necessary conditions for maximizing system utility functions that are increasing in the split rate vector. In the next section, the term *optimal beamforming parameterization* is introduced to denote parameterization of the optimal beamforming solution to RSMA performance maximization problem and used to reduce search space and complexity [29], [30].

The proposed idea is based on the connection between performance maximization and power minimization problems. To summarize briefly, for a given power constraint P_{tot}, if the optimal SINRs of the performance maximization problem are used as SINR thresholds of the power minimization problem, the optimal total power of the power minimization problem becomes P_{tot} and the same beamforming vectors solve both problems. Therefore, both problems can be said to have the same *structure* for their optimal beamforming vectors, and this structure is obtained as a parameterization of the beamforming directions based on the Lagrange multipliers of the power minimization problem.

The parameterization described above can be used to show that optimal beamforming provides a trade-off between maximizing a stream's SNR and reducing its interference on other streams. The idea is inspired from the relation between performance maximization and power minimization problems in SDMA where it is shown in [30] that minimum mean square error (MMSE) precoding is optimal in its beamforming structure for SDMA performance maximization. In the following, this concept is generalized to RSMA by proving that the optimal beamforming vectors that solve the performance maximization problem and the power minimization problem have the same structure.

The parameterization for multi-group multicasting beamforming of SDMA is obtained both for max-min rate optimization [31] and for a generalized objective function [32]. However, as there is no splitting of rates in SDMA, the streams are shared between disjoint groups of users in [31] and [32], while in RSMA there is coupling between streams, which makes the problem more complicated. The parameterization for optimal beamforming of RSMA is also obtained in [33] for WSR maximization of 1-layer RS. In this paper, the problem of obtaining optimal beamforming parameterization for RSMA in systems with full column rank channel matrices is addressed for the general form of RSMA with more than one common stream and a wider range of objective functions including but not limited to WSR maximization.

To design the beamforming vectors using the parameterization, an *optimal beamforming parameterization-based precoding design (OBPPD)* algorithm is proposed, that determines parameter values specifying beamforming directions and then allocates powers to streams for fixed beamforming directions. For determining beamforming direction parameters in OBPPD, the *duality-based directional beamforming (DDB)* algorithm is proposed that increases the objective function by increasing the split rate vector and checking feasibility by solving a linear problem at each step. The streams' powers in OBPPD are then found by solving a performance maximization problem for fixed beamforming directions. In general, power allocation for RSMA and arbitrary performance objective functions is not a convex problem. However, it may be expressed as a difference of convex functions, i.e., DC programming, and approximated iteratively by convex approximation. In addition, global optimality for the power allocation problem can be obtained by mixed monotonic programming (MMP) introduced in [34].

While the beamforming direction parameterization obtained is not optimal if the channel matrix is not of full column rank, the OBPPD algorithm is still applicable as a low complexity suboptimal beamforming design. The OBPPD precoding design approach reduces complexity significantly in comparison to existing precoding design approaches such as CCCP [22], which performs performance maximization with respect to covariance matrices. In contrast, in the proposed approach, beamforming directions are obtained from explicit expressions. The proposed power allocation solution also does not depend on numbers of antennas which lowers complexity, yet simulation results reveal that the performance of the proposed OBPPD is close to that of CCCP [22].

Power allocation problems for particular system utility functions are shown to be convex for the important special case where i) users do not split their messages and ii) each user's message can be decoded at a subset of users, which corresponds to a partial decoding of interference. We term this special case of RSMA streams *no rate-splitting, but partial interference decoding (NoRS-PID)*, and establish that SDMA and NOMA stream decoding are NoRS-PID special cases. As is previously known and confirmed by simulations, NOMA cannot effectively exploit the spatial domain's multiple antennas. By providing a trade-off between decoding interference and treating interference as noise, NoRS-PID is shown to outperform NOMA. In overloaded MU-MIMO systems, NoRS-PID is also shown to outperform SDMA. Based on the analytical results obtained, NoRS-PID power allocation can be expressed in convex form for three of the four performance objective functions listed earlier. The main contributions are summarized as follows:

- It is proved that the optimal beamforming vectors that solve the performance maximization and power minimization problems in RSMA have the same structure.
- A parameterization is obtained for optimal beamforming directions for RSMA performance maximization for systems with full column rank channel matrices and for performance evaluation metrics that are an increasing function of split user rates. The proposed OBPPD algorithm determines parameter values and beamforming directions from the obtained parameterization.
- The power allocation solution for RSMA performance maximization is approximated by DC programming for a generalized performance evaluation function.
- A new RSMA scheme, NoRS-PID, is introduced whose streams subsume SDMA and NOMA streams as special cases and whose power allocation is shown to be convex and easily solved for particular objective functions.

The rest of the paper is organized as follows: Section II specifies the system model. The parameterization for the optimal beamforming directions of the performance maximization problem is presented in Section III. In Section IV, RSMA beamforming vectors are found by setting direction parameter values and allocating powers. Section V presents simulation results, and Section VI provides conclusions.

Notation: Matrices and vectors are denoted by bold upper and lower-case letters, respectively. The inequality of vectors is component-wise. Hence, $\mathbf{a} > \mathbf{b}$ (or $\mathbf{a} \geq \mathbf{b}$) for vectors $\mathbf{a} = [a_1, \ldots, a_n]^T$ and $\mathbf{b} = [b_1, \ldots, b_n]^T$ denotes $a_i > b_i$ (or $a_i \geq b_i$) for all $i = 1, \ldots, N$. Sets are represented by calligraphic font $\mathcal{I}$, and $|\mathcal{I}|$ is the cardinality of $\mathcal{I}$. Relative complement of sets $\mathcal{J}$ and $\mathcal{I}$ is denoted by $\mathcal{I} \backslash \mathcal{J}$. Tuples are represented by serif font I. $|a|$, $\|\mathbf{a}\|_1$, and $\|\mathbf{a}\|_2$, respectively, represent absolute value, 1-norm, and Euclidean norm. $(\mathbf{A})^T$, $(\mathbf{A})^H$, and $\mathrm{Tr}(\mathbf{A})$ indicate transpose, conjugate-transpose, and trace, respectively. Positive-definite matrix $\mathbf{A}$ is denoted by $\mathbf{A} \succeq 0$, and $\mathbf{u}_{\max}(\mathbf{A})$ denotes the dominant eigenvector of matrix $\mathbf{A}$. $\mathbb{R}$ and $\mathbb{C}$ represent real and complex fields. $\mathcal{CN}(.,.)$ denotes a multi-variate circularly symmetric complex Gaussian distribution. $\delta_{(C)}$ is the indicator function, which is one when condition C is true, and zero otherwise. Finally, $\prod(.)$ represents scalar product.

II. SYSTEM MODEL

Consider a multi-user MISO system with M-antenna BS and K single antenna users, where $\mathcal{K} = \{1, \ldots, K\}$ is the set of users' indices. The channel between BS and users is denoted by $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K] \in \mathbb{C}^{M \times K}$, where $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$ is user k's channel vector. The quantity $\mathbf{H}_{\mathsf{I}}$ denotes a matrix consisting of channel vectors corresponding to the user indices in tuple l, e.g., for tuple $\mathsf{l} = (2, 1)$, $\mathbf{H}_{\mathsf{l}} = [\mathbf{h}_2, \mathbf{h}_1]$. As the main objective is to investigate novel precoding structures for RSMA and their optimality, CSIT is considered perfect here. Effects of imperfect CSIT can be examined through robust optimization or by simulation [35]. Consider an RSMA system where the BS splits users' messages and combines partial messages into streams shared between a subset of users. The BS sends N streams to the users such that stream i, $s_i, \forall i = 1, \ldots, N$, contains user messages in set $\mathcal{I}_i$ and must be decoded at all users in this set and treated as noise encountered by other users k, $\forall k \in \mathcal{K} \backslash \mathcal{I}_i$. We refer to $\mathcal{I}_i$ as a superuser, which contains $N_i = |\mathcal{I}_i|$ users. The set of superusers corresponding to the streams is represented by $\mathcal{G}$, where $|\mathcal{G}| = N$. An example for the case of three users is $\mathcal{G} = \{\{1\}, \{2\}, \{3\}, \{1, 2, 3\}\}$, which is a subset of the power set of user indices. The tuple of users in the ith superuser is denoted by l_i such that the index of uth user in the ith tuple is represented by $\mathsf{l}_i(u) = k \in \mathcal{I}_i, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$. Conversely, to indicate the index, u, of user k in the ith tuple, we define the notation $\bar{\mathsf{l}}_i(k) = u$. The BS sends a superposition of streams to the users, and the received signal at user k can be expressed as

$$y_k = \mathbf{h}_k^H \sum_{i=1}^{N} \mathbf{q}_i s_i + n_k, \quad \forall k \in \mathcal{K}, \tag{1}$$

where $\mathbf{q}_i \in \mathbb{C}^{M\times 1}, \forall i = 1, \ldots, N$ is the beamforming vector of the ith stream, and $n_k \sim \mathcal{CN}(0, 1)$ is the additive noise at user k. The beamforming vector corresponding to stream i is defined as $\mathbf{q}_i = \sqrt{p_i}\,\bar{\mathbf{q}}_i$, where p_i and $\bar{\mathbf{q}}_i$ denote the power and beamforming direction of the ith stream, respectively, where $\left\|\bar{\mathbf{q}}_i\right\|_2 = 1$. The beamforming vectors are represented by matrix $\mathbf{T} = [\mathbf{q}_1, \ldots, \mathbf{q}_N] \in \mathbb{C}^{M\times N}$.

Streams are decoded by SIC with decoding order chosen such that streams shared among more users must be decoded sooner than those shared by fewer users. Assume that streams are arranged such that for $N_i \geq N_j, \forall i < j \in \{1, 2, \ldots, N\}$. From [10], one possible decoding order is by increasing order of streams' indices, i.e., $\pi : s_1 \to s_2 \to \ldots s_N$. The signal observed in decoding the ith stream at user k is therefore

$$y_{i,k} = \mathbf{h}_k^H \left(\mathbf{q}_i s_i + \sum_{j=i+1}^{N} \mathbf{q}_j s_j + \sum_{j=1}^{i-1} \mathbf{q}_j s_j \delta_{(k \notin \mathcal{I}_j)}\right) + n_k, \forall k \in \mathcal{I}_i, \forall i = 1, \ldots, N \tag{2}$$

where streams that are decoded before s_i and contain part of user k's message, i.e., $\left\{s_j : k \in \mathcal{I}_j, \forall j = 1, \ldots, i-1\right\}$, are decoded and subtracted from the observed signal. Suppose that streams are zero mean, unit variance, and independent. From (2), the SINR in decoding the ith stream at the uth user of this stream ($\mathsf{l}_i(u) = k$) can be expressed as

$$\gamma_{i,u} = \frac{\mathbf{h}_{\mathsf{l}_i(u)}^H \mathbf{q}_i \mathbf{q}_i^H \mathbf{h}_{\mathsf{l}_i(u)}}{\mathbf{h}_{\mathsf{l}_i(u)}^H \left(\sum_{j=i+1}^{N} \mathbf{q}_j \mathbf{q}_j^H + \sum_{j=1}^{i-1} \mathbf{q}_j \mathbf{q}_j^H \delta_{(\mathsf{l}_i(u) \notin \mathcal{I}_j)}\right) \mathbf{h}_{\mathsf{l}_i(u)} + 1}, \quad \forall u = 1, \ldots, N_i, \quad \forall i = 1, \ldots, N. \tag{3}$$

The stream s_i must be successfully decoded by any user in $\mathcal{I}_i$. Thus, the achievable rate of the ith stream is as follows:

$$\begin{aligned} r_i &= \min_{u=1,\ldots,N_i} \log_2\left(1 + \gamma_{i,u}\right) \\ &= \log_2\left(1 + \min_{u=1,\ldots,N_i} \gamma_{i,u}\right), \quad \forall i = 1, \ldots, N. \end{aligned} \tag{4}$$

From (4), the SINR of the ith stream can be defined as

$$\gamma_i = \min_{u=1,\ldots,N_i} \gamma_{i,u}, \quad \forall i = 1, \ldots, N. \tag{5}$$

Let $r_{i,u}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$, be the rate allocated to the uth user of stream i, and $\mathbf{r}_i = \left[r_{i,1}, \ldots, r_{i,N_i}\right]^T, \forall i = 1, \ldots, N$, be the rate vector of split messages in stream i. From (4) and (5), the following inequality must be satisfied:

$$\|\mathbf{r}_i\|_1 = \sum_{u=1}^{N_i} r_{i,u} \leq r_i = \log_2\left(1 + \gamma_i\right), \quad \forall i = 1, \ldots, N. \tag{6}$$

The aggregate rate of user k, $\forall k = 1, \ldots, K$ can be expressed as a summation of its split rates as

$$\begin{aligned} R_k &= \sum_{i=1}^{N} \sum_{u=1}^{N_i} r_{i,u} \delta_{(\mathsf{l}_i(u)=k)} \\ &= [\mathbf{A}]_{:,k}^T \mathbf{r}, \quad [\mathbf{A}]_{\sum_{j=1}^{i-1} N_j + u, k} = \delta_{(\mathsf{l}_i(u)=k)}, \end{aligned} \tag{7}$$

$$\forall u = 1, \ldots, N_i, \quad \forall i = 1, \ldots, N, \quad \forall k \in \mathcal{K}, \tag{8}$$

where $\mathbf{r} = \left[\mathbf{r}_1^T, \ldots, \mathbf{r}_N^T\right]^T$ is the rate vector of split messages, matrix $\mathbf{A}$ is of size $\sum_{i=1}^{N} N_i \times K$, and $[\mathbf{A}]_{:,k}$ is the kth column of $\mathbf{A}$. Here, the rates of streams or partial messages in a stream are indicated by lower case letter "r", while the aggregate rate of users are indicated by capital letter "R".

The objective is to find the structure of optimal beamforming vectors in RSMA maximizing a system utility function $f(\mathbf{r})$ that is strictly increasing in split rates, i.e., for $\bar{\mathbf{r}} > \tilde{\mathbf{r}}$, $f(\bar{\mathbf{r}}) > f(\tilde{\mathbf{r}})$, which can be expressed as Problem $\mathcal{P}1$:

$$\mathcal{P}1: \underset{\substack{\mathbf{r}, \mathbf{q}_i, \\ \forall i=1,\ldots,N}}{\text{maximize}} \; f(\mathbf{r}) \tag{9a}$$

$$\text{subject to} \quad \|\mathbf{r}_i\|_1 \leq \log_2\left(1 + \gamma_i\right), \quad \forall i = 1, \ldots, N, \tag{9b}$$

$$\sum_{i=1}^{N} \|\mathbf{q}_i\|_2^2 \leq P_{\text{tot}}, \tag{9c}$$

where the constraint (9b) is in accordance with (6), which implies that the aggregate rate of split messages in a stream must not be greater than the rate of that stream. The strictly increasing condition for the system utility function is not a significant restriction. For example, in Section IV, several well-known objective functions $f(\cdot)$ are considered. In general, the strictly increasing condition is satisfied by the objective functions such that their maximum is necessarily on the Pareto boundary of the rate region. An example of a function that cannot satisfy this condition is energy efficiency (EE), which is usually expressed as a function increasing with rate in the numerator and total power in the denominator. As shown in Lemma 1, the higher the rate, the higher the required power. Hence, EE is neither increasing nor decreasing with the split rate. From the fact that $f(\cdot)$ is strictly increasing in split rates, it is straightforward to show that the constraints (9b) must be satisfied with equality at the global optimum of $\mathcal{P}1$. In addition, as streams' SINRs are increasing with total power (Appendix A, Lemma 1) and the objective function is increasing with streams' SINRs (Appendix C, Lemma 3), the total power constraint (9c) is also satisfied with equality at the optimal point. The notation $f_o = \mathcal{P}1\,(P_{\text{tot}})$ is equivalent to the following statement: for the total power limit P_{tot} in (9c), the optimal value of Problem $\mathcal{P}1$ is f_o.

By substituting the beamforming vectors of $\mathbf{T}$ in (3) and (5), suppose that the streams' SINR vector $\gamma = [\gamma_1, \ldots, \gamma_N]^T$ is achieved, denoted by $\gamma = G(\mathbf{T})$. By fixing beamforming matrix $\mathbf{T}$ in Problem $\mathcal{P}1\,(P_{\text{tot}})$ the following problem is obtained:

$$\mathcal{R}:\underset{\mathbf{r}}{\text{maximize}}\, f(\mathbf{r}) \tag{10a}$$

$$\text{subject to} \quad \|\mathbf{r}_i\|_1 \leq \log_2(1+\gamma_i), \quad \forall i = 1, \ldots, N, \tag{10b}$$

which is denoted by $\mathcal{R}(\gamma)$ such that $\gamma = G(\mathbf{T})$. Assuming that the optimal beamforming matrix of Problem $\mathcal{P}1\,(P_{\text{tot}})$ is $\mathbf{T}_o$, the optimal split rate vector of Problem $\mathcal{P}1\,(P_{\text{tot}})$ is the solution to Problem $\mathcal{R}(G(\mathbf{T}_o))$.

In an analogous manner to the case of SDMA [30], [36], we show that the solution to $\mathcal{P}1$ for the case of RSMA has the same structure as the power minimization solution, which is formulated as

$$\mathcal{P}2:\underset{\mathbf{q}_i, \forall i=1,\ldots,N}{\text{minimize}}\; \textstyle\sum_{i=1}^{N} \|\mathbf{q}_i\|_2^2 \tag{11a}$$

$$\text{subject to} \quad \gamma_i \geq \overline{\gamma}_i, \quad \forall i = 1, \ldots, N, \tag{11b}$$

where $\overline{\gamma} = \left[\overline{\gamma}_1, \ldots, \overline{\gamma}_N\right]^T$ denotes the quality of service (QoS), which is a vector comprised of each stream's QoS SINR. The notation $P_o = \mathcal{P}2\,(\overline{\gamma})$ is used to indicate that for the QoS vector $\overline{\gamma}$ in (11b), the optimal value of Problem $\mathcal{P}2$, is P_o. All QoS constraints in (11b) must be satisfied with equality at the optimal point of Problem $\mathcal{P}2\,(\overline{\gamma})$, because if by contradiction, the ith QoS constraint in (11b) is not active at the optimal point, we can always find a constant, c, $0 < c < 1$ that $c\mathbf{q}_i$ satisfies the ith QoS constraint, which reduces the total power. This is in contradiction with the optimality of the point. Note that replacing $\mathbf{q}_i$ by $c\mathbf{q}_i$ does not violate other QoS constraints, because according to (3), the lower the ith stream power, the lower its interference on other streams, resulting in higher SINRs for other streams.

Problems $\mathcal{P}1$ and $\mathcal{P}2$ are related by Theorem 4, Appendix B, as follows: if we substitute the minimum required power, P, obtained by Problem $\mathcal{P}2$ in the total power limit of Problem $\mathcal{P}1$, the same beamforming vectors that solve problem $\mathcal{P}2$ also solve Problem $\mathcal{P}1$. Problems $\mathcal{P}1$ and $\mathcal{P}2$ are also related by the fact that if we substitute the optimal SINR vector obtained by Problem $\mathcal{P}1$, i.e., γ, in the QoS constraints of Problem $\mathcal{P}2$, the same beamforming vectors that solve Problem $\mathcal{P}1$ also solve Problem $\mathcal{P}2\,(\gamma)$. This relation is expressed by Theorem 5, Appendix C.

According to Theorems 4 and 5, by solving either Problem $\mathcal{P}1$ or $\mathcal{P}2$, the solution to the other problem is also obtained. However, this statement comes with the following qualifications: (i) to find the solution to Problem $\mathcal{P}1$ by solving Problem $\mathcal{P}2$, the optimal streams' SINR vector of Problem $\mathcal{P}1$ must be provided as input to Problem $\mathcal{P}2$, which are not known a priori, and (ii) to obtain the solution to Problem $\mathcal{P}2$ by solving Problem $\mathcal{P}1$, the total power limit, i.e., the optimal value of Problem $\mathcal{P}2$ must be provided as input to $\mathcal{P}1$, which is unknown before solving $\mathcal{P}2$.

Notwithstanding the above two qualifications, from Theorems 4 and 5 we conclude that both problems have the same structure for their optimal beamforming vectors. In Section III, by analyzing the dual problem of $\mathcal{P}2$, a parameterization is derived for the optimal beamforming directions of performance maximization problem $\mathcal{P}1$ in systems with full column rank channel matrices, which reduces search space and complexity for beamforming design. Having a parameterization for the optimal beamforming directions, beamforming vectors are designed in Section IV to approximate the solution to $\mathcal{P}1$ by first determining parameter values and then by solving the power allocation problem. Simulation results in Section V show that the performance of the proposed beamforming design is close to that of an existing approach that approximates the optimal solution but with significant complexity reduction.

III. OPTIMAL BEAMFORMING PARAMETERIZATION

From the relation between $\mathcal{P}1$ and $\mathcal{P}2$, the optimal beamforming parameterizations of both problems are the same. Therefore, we start by analyzing the power minimization problem in $\mathcal{P}2$. From (3) and (5), Problem $\mathcal{P}2$ is equivalent to

$$\underset{\mathbf{q}_i, \forall i=1,\ldots,N}{\text{minimize}}\; \textstyle\sum_{i=1}^{N} \mathbf{q}_i^H \mathbf{q}_i \tag{12a}$$

$$\begin{aligned}\text{subject to} \quad & -\frac{1}{\overline{\gamma}_i}\mathbf{h}_{\mathsf{I}_i(u)}^H \mathbf{q}_i \mathbf{q}_i^H \mathbf{h}_{\mathsf{I}_i(u)} + \textstyle\sum_{j=i+1}^{N} \mathbf{h}_{\mathsf{I}_i(u)}^H \mathbf{q}_j \mathbf{q}_j^H \mathbf{h}_{\mathsf{I}_i(u)} \\ & + \textstyle\sum_{j=1}^{i-1} \mathbf{h}_{\mathsf{I}_i(u)}^H \mathbf{q}_j \mathbf{q}_j^H \mathbf{h}_{\mathsf{I}_i(u)} \delta_{(\mathsf{I}_i(u) \notin \mathcal{I}_j)} + 1 \leq 0, \\ & \forall u = 1, \ldots, N_i, \quad \forall i = 1, \ldots, N. \end{aligned} \tag{12b}$$

According to [30], the power minimization problem for SDMA can be expressed as second-order cone programming (SOCP). By applying Slater's condition, it is proven that strong duality and KKT conditions hold for SDMA. However, due to RSMA's common streams, Problem (12) is not convex. As KKT conditions are useful in obtaining the structure of an optimal solution, Theorem 1 provides a condition on whether the KKT conditions hold at any local minimum of the RSMA power minimization problem.

Theorem 1: If channel matrix $\mathbf{H}$ *is of full column rank, the KKT conditions hold at any local minimum of power minimization problem (12).*

Proof: See Appendix D. □

Remark 1: Theorem 1 is applicable to the under-loaded case where $M \geq K$*, since in practice channel vectors are unlikely to be linearly dependent.*

Assuming that $\mathbf{H}$ is of full column rank, any local optimum and as a result the global optimum of (12) must satisfy KKT conditions, which are provided by Theorem 2.

Theorem 2: The KKT conditions of Problem (12) at any local optimum $\mathbf{q}_i^o, \forall i = 1, \ldots, N$, *are given by*

$$(12b), \quad \forall u = 1, \ldots, N_i, \quad \forall i = 1, \ldots, N, \tag{13}$$

$$\lambda_{i,u} \geq 0, \quad \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N, \tag{14}$$

$$\max_{\mathbf{q}_i} \frac{\mathbf{q}_i^H \mathbf{A}_i \mathbf{q}_i}{\mathbf{q}_i^H \mathbf{B}_i \mathbf{q}_i} \leq \overline{\gamma}_i, \quad \forall i = 1, \ldots, N, \tag{15}$$

$$\left(\mathbf{B}_i - \frac{1}{\overline{\gamma}_i}\mathbf{A}_i\right)\mathbf{q}_i^o = 0, \quad \forall i = 1, \ldots, N, \tag{16}$$

$$\lambda_{i,u}\,(\text{left of }(12b)) = 0, \quad \forall u = 1, \ldots, N_i, \quad \forall i = 1, \ldots, N, \tag{17}$$

where $\lambda_{i,u}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$ *are the Lagrange multipliers of Problem (12), and* $M \times M$ *matrices,* $\forall i = 1, \ldots, N$,

$$\mathbf{A}_i = \mathbf{H}_{\mathsf{l}_i} \Sigma_i \mathbf{H}_{\mathsf{l}_i}^H, \tag{18}$$

$$\mathbf{B}_i = \mathbf{I}_M + \sum_{j=1}^{i-1} \mathbf{H}_{\mathsf{l}_j} \Sigma_j \mathbf{H}_{\mathsf{l}_j}^H + \sum_{j=i+1}^{N} \mathbf{H}_{\mathsf{l}_j} \mathbf{U}_{j,i} \Sigma_j \mathbf{H}_{\mathsf{l}_j}^H, \tag{19}$$

where Σ_i *and* $\mathbf{U}_{i,j}$ *are each diagonal matrices of size* $N_i \times N_i$ *with entries:*

$$[\Sigma_i]_{u,u} = \lambda_{i,u}, \quad \forall u = 1, \ldots, N_i, \quad \forall i = 1, \ldots, N, \tag{20}$$

$$\left[\mathbf{U}_{i,j}\right]_{u,u} = \delta_{(\mathsf{l}_i(u) \notin \mathcal{I}_j)}, \quad \forall u = 1, \ldots, N_i, \quad \forall i, j = 1, \ldots, N. \tag{21}$$

Proof: See Appendix E. □

The constraints in (15) are in the form of generalized Rayleigh quotient maximization. Based on the KKT conditions in (15) and (16), a parameterization for the optimal beamforming directions of the power minimization problem are obtained next. Multiplying $\left(\mathbf{q}_i^o\right)^H$ by the stationary condition (16), the expression $\frac{(\mathbf{q}_i^o)^H \mathbf{A}_i \mathbf{q}_i^o}{(\mathbf{q}_i^o)^H \mathbf{B}_i \mathbf{q}_i^o} = \overline{\gamma}_i, \forall i = 1, \ldots, N$, is derived. The maximum of the generalized Rayleigh quotient in (15) is obtained for any local optimum of Problem (12). Hence, the parameterization for the optimal beamforming directions of Problem (12) can be obtained by solving the generalized Rayleigh quotient maximization in (15):

$$\max_{\mathbf{q}_i} \frac{\mathbf{q}_i^H \mathbf{A}_i \mathbf{q}_i}{\mathbf{q}_i^H \mathbf{B}_i \mathbf{q}_i} = \max_{\mathbf{v}_i} \frac{\mathbf{v}_i^H \mathbf{B}_i^{-\frac{H}{2}} \mathbf{A}_i \mathbf{B}_i^{-\frac{1}{2}} \mathbf{v}_i}{\mathbf{v}_i^H \mathbf{v}_i}, \quad \forall i = 1, \ldots, N, \tag{22}$$

where the matrix $\mathbf{B}_i$ is Hermitian and expressible as $\mathbf{B}_i = \mathbf{B}_i^{\frac{H}{2}} \mathbf{B}_i^{\frac{1}{2}}$. In (22), the vector $\mathbf{v}_i = \mathbf{B}_i^{\frac{1}{2}} \mathbf{q}_i$, and the generalized Rayleigh quotient maximization becomes Rayleigh quotient maximization with the following solution:

$$\mathbf{v}_i = v_i \mathbf{u}_{\max}\left(\mathbf{B}_i^{-\frac{H}{2}} \mathbf{A}_i \mathbf{B}_i^{-\frac{1}{2}}\right), \quad \forall i = 1, \ldots, N, \tag{23}$$

where v_i is a scalar. From (23) and the definition of $\mathbf{v}_i$, the parameterization for the optimal beamforming direction of Problem (12) as a function of Lagrangian multipliers is

$$\overline{\mathbf{q}}_i = \frac{\mathbf{B}_i^{-\frac{1}{2}} \mathbf{u}_{\max}\left(\mathbf{B}_i^{-\frac{H}{2}} \mathbf{A}_i \mathbf{B}_i^{-\frac{1}{2}}\right)}{\left\|\mathbf{B}_i^{-\frac{1}{2}} \mathbf{u}_{\max}\left(\mathbf{B}_i^{-\frac{H}{2}} \mathbf{A}_i \mathbf{B}_i^{-\frac{1}{2}}\right)\right\|} \; \forall i = 1, \ldots, N. \tag{24}$$

Theorem 3: The global optimum of RSMA performance maximization problem $\mathcal{P}1$ *for a full column rank* $\mathbf{H}$ *is achieved by the parameterization (24) for some non-negative parameters* $\lambda_{i,u}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$ *and solving the rate and power allocation problem in the following:*

$$\underset{\mathbf{p} \geq \mathbf{0}, \mathbf{r} \geq \mathbf{0}}{\text{maximize}}\, f\left(\mathbf{r}\right) \tag{25a}$$

$$\text{subject to } \|\mathbf{r}_i\|_1 \leq \log_2\left(1 + \gamma_i\left(\overline{\mathbf{q}}_1, \ldots, \overline{\mathbf{q}}_N\right)\right), \quad \forall i = 1, \ldots, N, \tag{25b}$$

$$\|\mathbf{p}\|_1 \leq P_{tot}, \tag{25c}$$

which is obtained by fixing the beamforming directions in Problem $\mathcal{P}1$*. In Problem (25),* $\mathbf{p} = [p_1, \ldots, p_N]^T$ *is the vector of streams' powers,* $\gamma_i\left(\overline{\mathbf{q}}_1, \ldots, \overline{\mathbf{q}}_N\right)$ *in (25b) is the ith stream SINR* ($\forall i = 1, \ldots, N$) *obtained by substituting beamforming directions into (5):*

$$\gamma_i\left(\overline{\mathbf{q}}_1, \ldots, \overline{\mathbf{q}}_N\right) = \min_{u=1,\ldots,N_i} \gamma_{i,u}\left(\overline{\mathbf{q}}_1, \ldots, \overline{\mathbf{q}}_N\right), \tag{26}$$

$$\gamma_{i,u}\left(\overline{\mathbf{q}}_1, \ldots, \overline{\mathbf{q}}_N\right) = \frac{a_{i,u} p_i}{1 + \mathbf{d}_{i,u}^T \mathbf{p} - a_{i,u} p_i}, \quad \forall u = 1, \ldots, N_i, \tag{27}$$

where $\mathbf{d}_{i,u} \in \mathbb{R}_{\geq 0}^{N \times 1}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$, *and* $a_{i,u} \in \mathbb{R}_{\geq 0}$ *are defined, respectively, as:*

$$\left[\mathbf{d}_{i,u}\right]_j = \begin{cases} \left|\mathbf{h}_{\mathsf{l}_i(u)}^H \overline{\mathbf{q}}_j\right|^2 \delta_{(\mathsf{l}_i(u) \notin \mathcal{I}_j)}, & j = 1, \ldots, i-1, \\ \left|\mathbf{h}_{\mathsf{l}_i(u)}^H \overline{\mathbf{q}}_j\right|^2, & j = i, \ldots, N, \end{cases} \tag{28}$$

$$a_{i,u} = \left[\mathbf{d}_{i,u}\right]_i = \left|\mathbf{h}_{\mathsf{l}_i(u)}^H \overline{\mathbf{q}}_i\right|^2. \tag{29}$$

Proof: The proof follows from two facts: *(i)* based on Theorems 4 and 5, the optimal beamforming of Problems $\mathcal{P}1$ and $\mathcal{P}2$ have the same structure, and *(ii)* according to Theorems 1 and 2, that for any local optimum, including the global optimum of $\mathcal{P}2$ for a full column rank $\mathbf{H}$, there are Lagrange multiplier values for which the beamforming directions can be expressed by (24). □

Remark 2: Theorem 3 provides a parameterization for the solution to $\mathcal{P}1$ *that reduces the search space for the optimal solution. However, it does not determine the values of the parameters corresponding to the optimal solution to* $\mathcal{P}1$.

Remark 3: Solving $\mathcal{P}1$ *involves finding* MN *complex-valued parameters that determine* N *beamforming vectors for* M *antennas. In contrast, the parameterization provided by Theorem 3 requires determining* $\sum_{i=1}^{N} N_i$ *real parameters, and performing power allocation to determine* N *real-valued* *stream powers. From Theorem 3, it follows that the search complexity for the optimal solution to $\mathcal{P}1$ does not depend on the number of antennas M.*

Remark 4: For the overloaded case where $K > M$, the channel matrix is not of full column rank. While the parameterization in (24) still applies as an approximation, the local optima may not be KKT points which results in performance loss compared to that of optimal beamforming directions.

Matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ in (18) and (19), weight the effects of the ith stream beamforming on increasing stream SNR and managing interference on other streams, respectively. Therefore, the parameterization in (24) provides a trade-off between maximizing stream's SNR and reducing the stream's interference on other streams. Two special cases are *(i)* private streams that must be decoded only by one user, i.e., $\mathcal{I}_{N-K+k} = \{k\}, \forall k = 1, \ldots, K$, and *(ii)* the stream shared by all users $\mathcal{I}_1 = \mathcal{K}$, where stream indices are labelled based on the decoding rule that streams decoded by more users must be decoded first.

Private streams are considered as interference at all users except for the user intended. Therefore, (18) and (19) become:

$$\mathbf{A}_{N-K+k} = \lambda_{N-K+k,1}\mathbf{h}_k\mathbf{h}_k^H, \quad \forall k = 1, \ldots, K, \tag{30}$$

$$\mathbf{B}_{N-K+k} = \mathbf{I}_M + \mathbf{H}\Lambda\mathbf{H}^H - \lambda_{N-K+k,1}\mathbf{h}_k\mathbf{h}_k^H, \quad \forall k = 1, \ldots, K, \tag{31}$$

where Λ is a $K \times K$ diagonal matrix with elements $[\Lambda]_{k,k} = \sum_{i=1}^{N}\sum_{u=1}^{N_i}\lambda_{i,u}\delta_{(\mathsf{l}_i(u)=k)}, \forall k = 1, \ldots, K$. Hence, the beamforming directions for private streams are expressed as

$$\overline{\mathbf{q}}_{N-K+k} \overset{(a)}{=} \frac{\mathbf{B}_{N-K+k}^{-1}\mathbf{h}_k}{\left\|\mathbf{B}_{N-K+k}^{-1}\mathbf{h}_k\right\|} \overset{(b)}{=} \frac{\left(\mathbf{I}_M + \mathbf{H}\Lambda\mathbf{H}^H\right)^{-1}\mathbf{h}_k}{\left\|\left(\mathbf{I}_M + \mathbf{H}\Lambda\mathbf{H}^H\right)^{-1}\mathbf{h}_k\right\|}, \quad \forall k = 1, \ldots, K, \tag{32}$$

where in (32), (a) follows from the fact that $\mathbf{A}_{N-K+k}, \forall k = 1, \ldots, K$, are rank one, and (b) follows from the *Matrix Inversion Lemma* [37]. The parameterization in (32) is similar to the optimal beamforming parameterization of 1-layer RS WSR maximization derived for private streams in [33, (15b)]. From (32), it can be observed that the parameterization in (24) for SDMA, which only sends private streams, is actually *minimum mean square estimation (MMSE)* precoding [30]. If all Lagrange multipliers are equal, (32) becomes *regularized zero forcing (RZF)* precoding for SDMA. In [20], RSMA beamforming that nulls unintended interference is referred to as *interference nulling*, which is an extension of zero-forcing precoding to RSMA. By analogy between SDMA and RSMA, we term the beamforming parameterization in (24) for the case of equal Lagrange multipliers, i.e., $\lambda_{i,u} = \lambda, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$ as *regularized interference nulling (RIN)* precoding.

Another special case is the first stream $\mathcal{I}_1 = \mathcal{K}$, where $\mathbf{A}_1 = \mathbf{H}_{\mathsf{l}_1}\Sigma_1\mathbf{H}_{\mathsf{l}_1}^H$ and $\mathbf{B}_1 = \mathbf{I}_M$, resulting in

$$\overline{\mathbf{q}}_1 = \mathbf{u}_{\max}\left(\mathbf{H}_{\mathsf{l}_1}\Sigma_1\mathbf{H}_{\mathsf{l}_1}^H\right). \tag{33}$$

Since the common stream decoded by all users is not considered as interference by any users, its optimal beamforming direction must maximize the stream's SNR, $\gamma_1 = \min_{k=1,\ldots,K} \mathbf{q}_1^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{q}_1$, in order to maximize an increasing function of its rate. The SNR maximization of the stream shared by all users for a fixed power $\|\mathbf{q}_1\|^2 = p_1$ is formulated as

$$\underset{\mathbf{q}_1,\tau\geq 0}{\text{maximize}} \quad \tau \tag{34a}$$

$$\text{subject to} \quad \tau \leq \mathbf{q}_1^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{q}_1, \quad \forall k = 1, \ldots, K, \tag{34b}$$

$$\mathbf{q}_1^H\mathbf{q}_1 \leq p_1. \tag{34c}$$

Following a similar proof as in Appendix A for Theorem 1, it can be shown that KKT conditions hold at any local optimum of Problem (34) for a full-rank channel $\mathbf{H}$, and its KKT conditions result in (33), which also establishes the optimality of (33) in the sense of SNR maximization.

IV. PRECODING DESIGN USING OPTIMAL PARAMETERIZATION

The optimal beamforming parameterization described in the previous section is next applied to RSMA precoding design. By specifying Lagrange multipliers, beamforming directions are obtained by the expression (24). From Remark 2, optimal beamforming directions in (24) are parameterized in an explicit expression based on dominant eigenvectors. However, the parameter values that optimally solve Problem $\mathcal{P}1$ are unknown. In the following, an iterative algorithm is proposed to determine parameter values that approximate the solution to $\mathcal{P}1$. To complete the design of the downlink system, the streams' powers are obtained by solving Problem $\mathcal{P}1$ for fixed beamforming directions. This solution significantly reduces complexity, as the power allocation problem is made equivalent to the case of a single-antenna BS. This reduction in complexity is significant when the number of antennas is large.

The performance of each user is evaluated by its sum rate that is split into streams, i.e., $R_k = [\mathbf{A}]_{:,k}^T\mathbf{r}, \forall k = 1, \ldots, K$, and overall performance is evaluated by the system utility function. Since user performances are functions of split streams' rates, the system utility function $f(\cdot)$ can be defined as a function of $\mathbf{r}$, a vector of split rates. Four well-known utility functions of users' rates are: *i)* weighted arithmetic mean (WAM), or weighted sum rate (WSR), *ii)* weighted geometric mean (WGM), *iii)* weighted harmonic mean (WHM), *iv)* and weighted max-min (WMM) [29]. Maximizing sum rate maximizes aggregate user performance, while the max-min metric sacrifices aggregate user performance to maximize fairness. The remaining two metrics, geometric and harmonic means, are trade-offs between maximizing aggregate performance and maximizing fairness [29]. The system utility function can be further specified by assigning user weights such that user $k, \forall k = 1, \ldots, K$ has weight $w_k \geq 0$. These four RSMA system utility functions are expressed in the second column of Table 1, which are increasing in $\mathbf{r}$. The RSMA precoding design using Theorem 3 is termed *optimal beamforming parameterization-based precoding design (OBPPD)* and is summarized by the four steps in Algorithm 1. In Section IV-A, an algorithm is provided for determining the parameters in (24), which is the second step of OBPPD in Algorithm 1, and then in Section IV-B, Problem $\mathcal{P}1$ is solved for fixed beamforming directions to determine the beamforming vector powers, which is the fourth step in Algorithm 1.

TABLE 1. Examples of system utility functions for RSMA and NoRS-PID.

	$f(\mathbf{r})$ for RSMA $\left(R_k = [\mathbf{A}]^T_{:,k}\,\mathbf{r}\right)$	$f(\mathbf{r})$ for NoRS-PID $\left(f(\mathbf{r}) = g\left(\Vert\mathbf{r}_1\Vert_1, \ldots, \Vert\mathbf{r}_N\Vert_1\right)\right)$
WAM	$\sum_{k=1}^{K} w_k R_k$	$\sum_{i=1}^{N} w_{\mathrm{I}_i(\overline{u}_i)} \Vert\mathbf{r}_i\Vert_1$
WGM	$\prod_{k=1}^{K} R_k^{w_k}$	$\prod_{i=1}^{N} \left(\Vert\mathbf{r}_i\Vert_1\right)^{w_{\mathrm{I}_i(\overline{u}_i)}}$
WHM	$\left(\sum_{k=1}^{K} \frac{w_k}{R_k}\right)^{-1}$	$\left(\sum_{i=1}^{N} \frac{w_{\mathrm{I}_i(\overline{u}_i)}}{\Vert\mathbf{r}_i\Vert_1}\right)^{-1}$
WMM	$\min_{k=1,\ldots,K} \frac{R_k}{w_k}$	$\min_{i=1,\ldots,N} \frac{\Vert\mathbf{r}_i\Vert_1}{w_{\mathrm{I}_i(\overline{u}_i)}}$

Algorithm 1 Optimal Beamforming Parameterization-Based Precoding Design (OBPPD)

Inputs: Channel $\mathbf{H}$ and streams $s_i, \forall i = 1, \ldots, N$
Outputs: Beamforming vectors $\overline{\mathbf{q}}_i, \forall i = 1, \ldots, N$, and split rates $\mathbf{r}$

1 Specify $f(\mathbf{r})$ that is an increasing function of $\mathbf{r}$;
2 Specify parameters $\lambda_{i,u}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$, e.g. by RIN or DDB described in Section IV-A;
3 Find beamforming directions from (24) and (18)-(19);
4 Find streams' powers by solving Problem (25), e.g., by MMP [34] or by approximating the powers by DC programming [28].

A. DESIGNING BEAMFORMING DIRECTIONS

To determine Lagrange multiplier values, one obvious choice is equal values for all multipliers (RIN precoding). Since duality for the power minimization problem suggests $\sum_{i=1}^{N}\sum_{u=1}^{N_i}\lambda_{i,u} \leq P_{\mathrm{tot}}$, the Lagrange multipliers are chosen as $\lambda_{i,u} = \frac{P_{\mathrm{tot}}}{\sum_{i=1}^{N} N_i}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$. Since RIN precoding parameters are independent of the system utility function $f(\mathbf{r})$, precoding design accuracy is likely reduced compared to a procedure that takes $f(\mathbf{r})$ into account.

An alternative approach utilizes the relation between the performance maximization problem $\mathcal{P}1$ and the power minimization problem $\mathcal{P}2$. According to Theorem 5, assuming that the optimal streams' SINR vector of $\mathcal{P}1(P_{\mathrm{tot}})$ is γ_{o}, the same beamforming vectors solve both Problems $\mathcal{P}1(P_{\mathrm{tot}})$ and $\mathcal{P}2(\gamma_{\mathrm{o}})$. Exploiting the assumption that the system utility function $f(\mathbf{r})$ is an increasing function of $\mathbf{r}$, the idea is to incrementally increase the split rate vector $\mathbf{r}$ to increase $f(\mathbf{r})$ and approach the optimal value of Problem $\mathcal{P}1$. Having split rates, the streams' rates and SINRs are obtained from (6). The relation between $\mathcal{P}1$ and $\mathcal{P}2$ is used to check the feasibility of $\mathbf{r}$.

As the power constraint in Problem $\mathcal{P}1(P_{\mathrm{tot}})$ is active at the optimal point, the optimal SINRs of $\mathcal{P}1(P_{\mathrm{tot}})$ can be achieved with total power P_{tot}, and according to Theorem 5, $P_{\mathrm{tot}} = \mathcal{P}2(\gamma_{\mathrm{o}})$, where γ_{o} is the optimal SINR vector of $\mathcal{P}1(P_{\mathrm{tot}})$. Let $\gamma_{\overline{\mathrm{o}}}$ denote a non-optimal SINR vector of $\mathcal{P}1(P_{\mathrm{tot}})$. The vector $\gamma_{\overline{\mathrm{o}}}$ is either *(i)* feasible and suboptimal or *(ii)* infeasible. For case *(i)*, we have $\mathcal{P}2(\gamma_{\overline{\mathrm{o}}}) < P_{\mathrm{tot}}$, and for case *(ii)*, we have $\mathcal{P}2(\gamma_{\overline{\mathrm{o}}}) > P_{\mathrm{tot}}$. As Problem $\mathcal{P}2$ is nonconvex, we approximate $\mathcal{P}2(\gamma_{\overline{\mathrm{o}}})$ with its dual problem optimal value, i.e., the sum of Lagrange multipliers $\sum_{i=1}^{N}\sum_{u=1}^{N_i}\lambda_{i,u}$. From (59), Appendix E, the dual problem of $\mathcal{P}2(\gamma_{\overline{\mathrm{o}}})$ can be expressed as follows:

$$\underset{\substack{\lambda_{i,u}\geq 0,\\ \forall u=1,\ldots,N_i,\\ \forall i=1,\ldots,N}}{\text{maximize}} \ \sum_{i=1}^{N}\sum_{u=1}^{N_i}\lambda_{i,u} \tag{35a}$$

$$\text{subject to } \ \mathbf{B}_i - \left(1/\overline{\gamma}_i\right)\mathbf{A}_i \succeq 0, \forall i = 1, \ldots, N, \tag{35b}$$

which is a semidefinite programming (SDP) with linear matrix inequalities (LMIs) in (35b) and can be solved by MATLAB's cvx tools. Therefore, to check the feasibility of the rate vector and find the corresponding Lagrange multipliers, Problem (35) is solved. If $\sum_{i=1}^{N}\sum_{u=1}^{N_i}\lambda_{i,u} > P_{\mathrm{tot}}$, the rate vector $\mathbf{r}$ is infeasible in Problem $\mathcal{P}1(P_{\mathrm{tot}})$.[1]

To approach the maximum value of $f(\mathbf{r})$, the split rates are increased in the direction of the system utility function gradient such that $\mathbf{r} = \mathbf{r}^{\mathrm{old}} + \xi\nabla_{\mathbf{r}} f\left(\mathbf{r}^{\mathrm{old}}\right)$, where $\mathbf{r}^{\mathrm{old}}$ is the rate vector in previous iteration, and $\xi > 0$ controls the rate vector increment. After updating the rate vector, the SINR thresholds are updated from (6). If the result of solving Problem (35) with new SINR thresholds shows the rate vector to be infeasible, the rates and the increment factor ξ are decreased. The algorithm aims to drive $\left|\sum_{i=1}^{N}\sum_{u=1}^{N_i}\lambda_{i,u} - P_{\mathrm{tot}}\right|$ toward a fixed positive value over a few iterations. By increasing $\mathbf{r}$, the total power required to achieve $\mathbf{r}$ increases. Therefore, it is expected that $\sum_{i=1}^{N}\sum_{u=1}^{N_i}\lambda_{i,u}$ also increases at each iteration driving $\left|\sum_{i=1}^{N}\sum_{u=1}^{N_i}\lambda_{i,u} - P_{\mathrm{tot}}\right|$ toward the duality gap of Problem $\mathcal{P}2$. Therefore, as strong duality is not guaranteed for the power minimization problem, the algorithm may not converge to optimal parameter values, but instead to a fixed point, which, though suboptimum, still provides suitable candidates for the parameter values as shown in Section V. By substituting values of the Lagrange multipliers at the convergence point into (24), the beamforming directions are obtained. This approach is termed *duality-based directional beamforming (DDB)*, and the observations in Section V show

[1] As strong duality is not guaranteed for Problem $\mathcal{P}2$, the case $\sum_{i=1}^{N}\sum_{u=1}^{N_i}\lambda_{i,u} < P_{\mathrm{tot}}$ may correspond to a feasible or infeasible split rate vector. As feasibility cannot be checked without solving Problem $\mathcal{P}2$, an approximation is made to consider this case as corresponding to a feasible $\mathbf{r}$ in Problem $\mathcal{P}1(P_{\mathrm{tot}})$.

Algorithm 2 Duality-Based Directional Beamforming (DDB)

Inputs: Channel $\mathbf{H}$ and streams $s_i, \forall i = 1, \ldots, N$
Outputs: Beamforming directions $\overline{\mathbf{q}}_i, \forall i = 1, \ldots, N$
1 Set $\xi = 0.5$;
2 Initialize $\lambda_{i,u} = \frac{P_{\text{tot}}}{\sum_{i=1}^{N} N_i}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$;
3 Compute $\overline{\mathbf{q}}_i, \forall i = 1, \ldots, N$ from (24);
4 Allocate equal powers to streams $p_i = \frac{P_{\text{tot}}}{N}, \forall i = 1, \ldots, N$;
5 Compute SINRs from (5) and set them as SINR thresholds $\overline{\gamma}_i, \forall i = 1, \ldots, N$;
6 Compute streams' rates from (4) and set split rates as $r_{i,u} = \frac{r_i}{N_i}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$;
7 Solve problem (35) to obtain Lagrange multipliers $\lambda_{i,u}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$;
8 **while** *Not converged* **do**
9   Update rate vector as $\mathbf{r} = \mathbf{r}^{\text{old}} + \xi \nabla_{\mathbf{r}} f\left(\mathbf{r}^{\text{old}}\right)$;
10   Compute SINRs from (5) and set them as SINR thresholds $\overline{\gamma}_i, \forall i = 1, \ldots, N$;
11   Solve problem (35) to obtain Lagrange multipliers $\lambda_{i,u}, \forall u = 1, \ldots, N_i, \forall i = 1, \ldots, N$;
12   **if** $\sum_{i=1}^{N} \sum_{u=1}^{N_i} \lambda_{i,u} > P_{tot}$ **then**
13   Set rate vector as $\mathbf{r} = \mathbf{r}^{\text{old}}$;
14   Decrease ξ by a factor of 2;
15   **end**
16 **end**
17 Compute $\overline{\mathbf{q}}_i, \forall i = 1, \ldots, N$ from (24).

that DDB outperforms RIN precoding as expected, since Lagrange multipliers are chosen to maximize system utility function $f(\mathbf{r})$.

The procedure to obtain beamforming directions, DDB, is summarized as Algorithm 2. Lines 2-6 obtain an initial feasible rate vector $\mathbf{r}$, and the other steps are described above.

B. POWER ALLOCATION

Using the parameterization in (24) and the proposed methods to determine parameters, beamforming directions $\overline{\mathbf{q}}_i, \forall i = 1, \ldots, N$, are designed. The next precoding design step is power allocation, which is achieved by solving performance maximization $\mathcal{P}1$ for fixed beamforming directions as expressed in Problem (25). This problem is non-convex for most known objective functions, since for RSMA the rate of each user is the sum rate of its split messages and sum rate terms are non-convex [38]. In the following, a special case of RSMA is introduced that has a convex power allocation problem for certain system utility functions.

Consider the special case of RSMA where users do not split their rate, and as a result, there are $N = K$ streams, but each user's message can be decoded by more than one user. Similar to the general case of RSMA, the superusers $\mathcal{I}_i, \forall i = 1, \ldots, N$, are sets of users that must decode stream i. In the case of no rate splitting, the rate of stream i is assigned only to one of the users in $\mathcal{I}_i$. We refer to this special case of RSMA as *no RS, but partial interference decoding (NoRS-PID)*, which corresponds to the case that satisfies the following conditions:

- the number of users and streams are equal, i.e., $N = K$,
- the ith stream rate ($\forall i = 1, \ldots, N$) is allocated to one of the users in $\mathcal{I}_i$, i.e.,

$$r_{i,u}\delta_{(u \in \{\{1,\ldots,N_i\}\setminus\{\overline{u}_i\}\})} = 0, \tag{36}$$

  which results in $r_{i,u} = \|\mathbf{r}_i\|_1 \, \delta_{(u=\overline{u}_i)}, \forall i = 1, \ldots, N$, and
- users do not split their rate. Therefore, the rate of each user corresponds to one of the streams' rates, i.e., for each user $k \in \mathcal{K}$, there exists only one stream index $i \in \{1, \ldots, N\}$ such that $k = \mathrm{l}_i(\overline{u}_i)$.

Non-orthogonal multiple access streams are an instance of such a scheme. As shown in the simulation results, using NOMA streams cannot effectively exploit spatial degrees of freedom offered by multiple antennas. In contrast, NoRS-PID provides a trade-off between decoding interference and treating interference as noise and is able to outperform NOMA streams.

For NoRS-PID, the system utility function can be expressed as a function of streams' rates, i.e., $\|\mathbf{r}_i\|_1, \forall i = 1, \ldots, N$. The third column of Table 1 provides the system utility functions for NoRS-PID, and the power allocation problem for NoRS-PID can be formulated as

$$\underset{\mathbf{p}\geq\mathbf{0},\mathbf{r}\geq\mathbf{0}}{\text{maximize}} \; f(\mathbf{r}) = g\left(\|\mathbf{r}_1\|_1, \ldots, \|\mathbf{r}_N\|_1\right) \tag{37a}$$

$$\text{subject to} \quad \|\mathbf{r}_i\|_1 \leq \log_2\left(1 + \gamma_i\left(\overline{\mathbf{q}}_1, \ldots, \overline{\mathbf{q}}_N\right)\right), \quad \forall i = 1, \ldots, N, \tag{37b}$$

$$\|\mathbf{p}\|_1 \leq P_{\text{tot}}. \tag{37c}$$

Since $f(\mathbf{r})$ is an increasing function of $\mathbf{r}$, the constraints in (37b) must be satisfied with equality at the optimal point. Therefore, the constraints in (37b) can be merged with the objective function by substituting $\|\mathbf{r}_i\|_1, \forall i = 1, \ldots, N$, with $\log_2\left(1 + \gamma_i\left(\overline{\mathbf{q}}_1, \ldots, \overline{\mathbf{q}}_N\right)\right)$ in the objective function of Problem (37). The power allocation problem is expressed as

$$\underset{\mathbf{p}\geq\mathbf{0}}{\text{maximize}} \; g\Big(\log_2\left(1 + \gamma_1\left(\overline{\mathbf{q}}_1, \ldots, \overline{\mathbf{q}}_N\right)\right), \ldots, \log_2\left(1 + \gamma_N\left(\overline{\mathbf{q}}_1, \ldots, \overline{\mathbf{q}}_N\right)\right)\Big) \tag{38a}$$

$$\text{subject to} \quad \|\mathbf{p}\|_1 \leq P_{\text{tot}}. \tag{38b}$$

In contrast to Problem (25), the one-norm of rates does not appear in the constraints of Problem (38), which makes the power allocation problem tractable for certain system utility functions. In the following, the convex formulation for these power allocation problems in NoRS-PID is first obtained. Then, an algorithm based on the concave-convex procedure (CCCP) [28] is proposed to approximate the solution to the general power allocation problem in RSMA.

TABLE 2. Equivalent problems for maximizing system utility functions.

	RSMA: $\underset{\mathbf{r}\geq 0}{\text{maximize}} f(\mathbf{r})$	NoRS-PID: $\underset{\tilde{\gamma}_i\geq 0, \forall i=1,\ldots,N}{\text{maximize}} g\left(\exp(\tilde{\gamma}_1),\ldots,\exp(\tilde{\gamma}_N)\right)$
WAM	$\underset{\mathbf{r}\geq 0}{\max.} \sum_{k=1}^{K} w_k [\mathbf{A}]_{:,k}^T \mathbf{r}$	It cannot be expressed in a convex form by variable change.
WGM	$\underset{\mathbf{r}\geq 0}{\max.} \sum_{k=1}^{K} w_k \ln\left([\mathbf{A}]_{:,k}^T \mathbf{r}\right)$	$\underset{\tilde{\gamma}_i\geq 0, \forall i=1,\ldots,N}{\max.} \sum_{i=1}^{N} w_{\mathbf{l}_i(\bar{u}_i)} \tilde{\gamma}_i$
WHM	$\underset{\mathbf{r}\geq 0}{\min.} \sum_{k=1}^{K} \frac{w_k}{[\mathbf{A}]_{:,k}^T \mathbf{r}}$	$\underset{\tilde{\gamma}_i\geq 0, \forall i=1,\ldots,N}{\min.} \sum_{i=1}^{N} w_{\mathbf{l}_i(\bar{u}_i)} \exp(-\tilde{\gamma}_i)$
WMM	$\underset{\alpha\geq 0,\mathbf{r}\geq 0}{\max.} \alpha$ s. t. $\frac{[\mathbf{A}]_{:,k}^T \mathbf{r}}{w_k} \geq \alpha$, $\forall k=1,\ldots,K$	$\underset{\alpha\geq 0,\tilde{\gamma}_i\geq 0, \forall i=1,\ldots,N}{\max.} \alpha$ s. t. $\tilde{\gamma}_i - \ln\left(w_{\mathbf{l}_i(\bar{u}_i)}\right) \geq \alpha$, $\forall i=1,\ldots,N$

1) POWER ALLOCATION FOR NORS-PID

Power allocation for NoRS-PID can be formulated in a convex form for WGM, WHM, and WMM of users' rates. Defining auxiliary variables $\overline{\gamma}_i, \forall i = 1,\ldots,N$, and using epigraph form, Problem (38) can be expressed as

$$\underset{\substack{\mathbf{p}\geq\mathbf{0},\overline{\gamma}_i\geq 0,\\ \forall i=1,\ldots,N}}{\text{maximize}} \; g\left(\log_2\left(1+\overline{\gamma}_1\right),\ldots,\log_2\left(1+\overline{\gamma}_N\right)\right) \tag{39a}$$

$$\text{subject to} \;\; \gamma_{i,u} = \frac{a_{i,u}p_i}{1+\mathbf{d}_{i,u}^T\mathbf{p}-a_{i,u}p_i} \geq \overline{\gamma}_i, \quad \forall u = 1,\ldots,N_i, \quad \forall i = 1,\ldots,N, \tag{39b}$$

$$\|\mathbf{p}\|_1 \leq P_{\text{tot}}. \tag{39c}$$

To express Problem (39) in a convex form, the variable changes $p_i = \exp(\tilde{p}_i), \forall i = 1,\ldots,N$, and $\log_2\left(1+\overline{\gamma}_i\right) = \exp(\tilde{\gamma}_i), \forall i = 1,\ldots,N$, are required [29]. Therefore, Problem (39) can be formulated as

$$\underset{\substack{\tilde{\mathbf{p}}\geq\mathbf{0},\tilde{\gamma}_i\geq 0,\\ \forall i=1,\ldots,N}}{\text{maximize}} \; g\left(\exp(\tilde{\gamma}_1),\ldots,\exp(\tilde{\gamma}_1)\right) \tag{40a}$$

$$\text{subject to} \;\; \ln\left(2^{\exp(\tilde{\gamma}_i)}-1\right) + \ln\left(\frac{\exp(-\tilde{p}_i)}{a_{i,u}} + \sum_{\substack{j=1,\\ j\neq i}}^{N} \frac{[\mathbf{d}_{i,u}]_j}{a_{i,u}} \exp\left(\tilde{p}_j-\tilde{p}_i\right)\right) \leq 0, \quad \forall u = 1,\ldots,N_i, \forall i = 1,\ldots,N, \tag{40b}$$

$$\sum_{i=1}^{N} \exp(\tilde{p}_i) \leq P_{\text{tot}}, \tag{40c}$$

The constraints of Problem (40) are convex, and if its objective function is a convex problem, Problem (40) is convex and can be solved by MATLAB's cvx tools.

Table 2 lists the equivalent problems for maximizing the three system utility functions mentioned above for NoRS-PID. All problems in this table are convex. In Table 2, the objective functions for WGM and WHM are obtained based on the fact that $\exp(\cdot)$ and $\log_2(\cdot)$ are increasing functions and maximizing an increasing function is equivalent to maximizing its argument. In addition, the objective function of WMM is obtained by using epigraph form and adding the auxiliary variable α.

It is worth noting that while Problem (40) is convex for certain system utility functions, the constraints in (40b) are not in forms that can be solved by cvx [39]. However, the power allocation problem in (40) can be solved using well known interior-point methods [39].

2) POWER ALLOCATION FOR GENERAL FORMS OF RSMA

As mentioned earlier, the RSMA power allocation is non-convex in most cases. In the following, an algorithm is proposed to approximate the solution to the power allocation problem for general forms of RSMA and for a general system utility function $f(\mathbf{r})$, which is increasing with respect to $\mathbf{r}$. By converting the minimum function in the ith constraint of (25b) to N_i constraints, Problem (25) can be expressed as follows:

$$\underset{\mathbf{p}\geq\mathbf{0},\mathbf{r}\geq\mathbf{0}}{\text{maximize}} \; f(\mathbf{r}) \tag{41a}$$

$$\text{subject to} \;\; \|\mathbf{r}_i\|_1 \leq \log_2\left(1+\frac{a_{i,u}p_i}{1+\mathbf{d}_{i,u}^T\mathbf{p}-a_{i,u}p_i}\right), \quad \forall u = 1,\ldots,N_i, \quad \forall i = 1,\ldots,N, \tag{41b}$$

$$\|\mathbf{p}\|_1 \leq P_{\text{tot}}, \tag{41c}$$

Problem (41) is non-convex, because of rational functions in (41b), but these constraints can be expressed as differences of convex functions, i.e., DC programming:

$$\underbrace{\|\mathbf{r}_i\|_1 - \log_2\left(1+\mathbf{d}_{i,u}^T\mathbf{p}\right)}_{\text{convex}} + \underbrace{\log_2\left(1+\mathbf{d}_{i,u}^T\mathbf{p}-a_{i,u}p_i\right)}_{\text{concave}} \leq 0, \quad \forall u = 1,\ldots,N_i, \quad \forall i = 1,\ldots,N. \tag{42}$$

Therefore, the solution to Problem (41) can be approximated based on the concave-convex procedure (CCCP) [22], [28]. Following the approach in [22] for a DC programming problem, Problem (41) solution can be approximated successively by solving a convex sub-problem, which is obtained by replacing the concave part in (42) with its linear approximation. The sub-problem in iteration $\kappa+1$ is expressed as follows:

$$\underset{\mathbf{p}\geq\mathbf{0},\mathbf{r}\geq\mathbf{0}}{\text{maximize}} \; f(\mathbf{r}) \tag{43a}$$

$$\text{subject to} \;\; \|\mathbf{r}_i\|_1 - \log_2\left(1+\mathbf{d}_{i,u}^T\mathbf{p}\right) + \log_2\left(1+\mathbf{d}_{i,u}^T\mathbf{p}^{(\kappa)}-a_{i,u}p_i^{(\kappa)}\right) + \frac{\mathbf{d}_{i,u}^T\left(\mathbf{p}-\mathbf{p}^{(\kappa)}\right)-a_{i,u}\left(p_i-p_i^{(\kappa)}\right)}{\left(1+\mathbf{d}_{i,u}^T\mathbf{p}^{(\kappa)}-a_{i,u}p_i^{(\kappa)}\right)\ln 2} \leq 0, \quad \forall u = 1,\ldots,N_i, \quad \forall i = 1,\ldots,N, \tag{43b}$$

$$\|\mathbf{p}\|_1 \leq P_{\text{tot}}, \tag{43c}$$

which can be solved by MATLAB's cvx tools. The convergence of the power allocation algorithm to a stationary point is proven in [28]. The equivalent objective functions for WAM, WGM, WHM, and WMM of users' rate such that the sub-problem (43) becomes a convex problem are listed in Table 2. Problem (41) can also be expressed as mixed monotonic programming (MMP) introduced in [34], and solved efficiently by the branch-and-bound (BB) solution to $\mathcal{P}1$. Hence, if the algorithm in [34] converges to the optimal beamforming directions of $\mathcal{P}1$, the overall beamforming design algorithm, termed OBPPD, converges to the global optimum of $\mathcal{P}1$. However, as mentioned earlier, the DDB algorithm may not converge to the optimum solution.

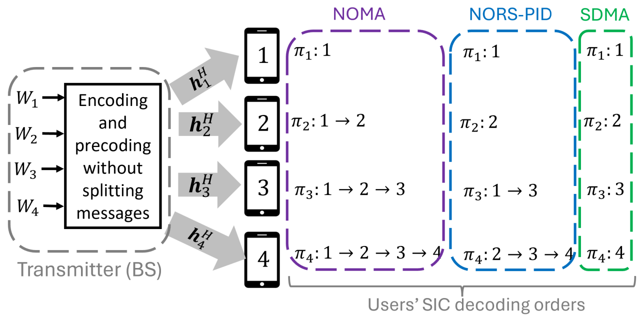


FIGURE 1. **An example of NOMA, NoRS-PID, and SDMA for 4 users.**

V. SIMULATION RESULTS

A. PERFORMANCE COMPARISON METHODOLOGY

The performance of RSMA using OBPPD precoding algorithm is evaluated for the four objective functions: sum rate, geometric mean, harmonic mean, and minimum user's rate. The OBPPD algorithm that uses RIN in the second step of Algorithm 1 is termed OBPPD-RIN, and it is termed OBPPD-DDB if it uses the DDB algorithm for parameter determination. In both cases, the DC programming described in Section IV-B is used for power allocation. The performance of the proposed solutions are compared with that of the CCCP [22, Section IV-A][2] and the SIN-MaxSNR [20] algorithms.

The CCCP algorithm [22], used as a benchmark approximation to the optimal solution to $\mathcal{P}1$, is formulated in DC programming form with respect to covariance matrices $\mathbf{Q}_i, \forall i = 1, \ldots, N$, instead of beamforming vectors $\mathbf{q}_i, \forall i = 1, \ldots, N$, as in (44a)–(44c), shown at the bottom of the next page.

A convex sub-problem is obtained by substituting the concave part of (44b) by its linear approximation. The algorithm, proven to converge to a stationary point [22], [28], is analyzed for both WAM and WMM of users' rates in [22].

As OBPPD provides a trade-off between maximizing streams' SNRs and reducing inter-stream interference, it is also compared with SIN-MaxSNR [20], an RSMA beamforming design under an interference nulling constraint (RSMA-IN) [20], a near optimal scheme for under-loaded regimes ($K \leq M$), that prevents interference to users that must not decode that stream.

For a fair comparison, it is assumed that all precoding schemes use the same set of streams with a fixed decoding order. For RSMA streams, 2-layer RS [10], [11] is generated with the user grouping algorithm [12, Algorithm 1], which has 3 levels of user grouping: K single-user groups at the first level, a group containing all users at the third level, and a set of disjoint user groups of size between 1 and K at the second level. An example is the set of groups $\mathcal{G} = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 2, 3\}\}$ corresponding to 2-layer RS for 3 users. According to [20], 2-layer RS is a form of hierarchical RS, which has unique SIC decoding order.

[2] The concave-convex procedure (CCCP) is a majorization-maximization method, but here "CCCP" refers to the algorithm in [22, Section IV-A].

The NOMA scheme is a special case of RSMA with $N = K$ streams, where the stream i must be decoded at user receivers in the set $\mathcal{I}_i^{\text{NOMA}} = \{i, i+1, \ldots, K\}$, and the ith stream's rate is allocated to the ith user, i.e., $r_i^{\text{NOMA}} = R_i^{\text{NOMA}}, \forall i = 1, \ldots, K$. For NOMA, a sub-optimal SIC decoding order, that is, decoding users' messages in increasing channel norm order is considered. Therefore, assuming $\|\mathbf{h}_l\|_2 \leq \|\mathbf{h}_k\|_2, \forall l \leq k, \forall l, k \in \{1, \ldots, K\}$, the NOMA SIC order is $\pi_{\text{NOMA}} : 1 \to \cdots \to K$.

The streams for NoRS-PID are generated out of NOMA's K streams, because both schemes do not split users' rates, and the rate of each stream is allocated only to one user. Unlike NOMA where user i must decode the interference of users $\{1, \ldots, i-1\}$, in NoRS-PID, user i may decode the interference of a only subset of these users. The NOMA streams are modified to generate NoRS-PID streams as follows: if user $j \in \{i+1, \ldots, K\}$ does not decode user i's message, index j is removed from $\mathcal{I}_i^{\text{NOMA}}$, i.e., the set of NoRS-PID streams is generated as

$$\mathcal{I}_i^{\text{NoRS-PID}} = \mathcal{I}_i^{\text{NOMA}} \backslash \mathcal{A}_i, \quad \forall i = 1, \ldots, K, \tag{45}$$

where $\mathcal{A}_i$ is a subset of users in $\{i+1, \ldots, K\}$ that do not decode the ith user's message. To find $\mathcal{A}_i, \forall i = 1, \ldots, K$, a channel similarity metric is used [19]. If channel similarity between user i and j is less than a threshold, β, user j treats the interference of user i as noise. The sets $\mathcal{A}_i, \forall i = 1, \ldots, N$ are generated as

$$\mathcal{A}_i = \left\{ j \in \{i+1, \ldots, K\} \middle| \frac{\left\| \mathbf{h}_j^H \mathbf{h}_i \right\|_2}{\left\| \mathbf{h}_j \right\|_2 \|\mathbf{h}_i\|_2} \leq \beta \right\}. \tag{46}$$

As the value of β increases, the number of users decoding the ith stream, i.e., $\left|\mathcal{I}_i^{\text{NoRS-PID}}\right|$, decreases. Decoding for SDMA corresponds to $\beta = 1$, where each user decodes a single stream, and NOMA decoding is obtained by $\beta = 0$. That is, β controls the trade-off between NOMA and SDMA. In the simulation results, the value of β is set to 0.5.

B. COMPLEXITY REDUCTION

Recall that OBPPD has two stages: *i)* setting parameters for beamforming directions, and *ii)* power allocation. The

OBPPD-RIN algorithm does not explicitly determine beamforming direction parameters so its complexity depends only on the power allocation process. On the other hand, the complexity of OBPPD-DDB mainly depends on the complexity of the DDB algorithm. Table 3 provides complexities of sub-problems in the iterative algorithms: CCCP [22], DDB, and RSMA power allocation by DC programming. All of these sub-problems are convex, and the complexity of solving a convex problem with m inequalities and n variables using interior point and log-barrier methods [39, Chapter 11] is $\mathcal{O}\left(m^{1.5}n^2\right)$. The interior-point method approximates the convex problem by the log-barrier method and obtains a convex problem that only has equality constraints, which can be solved by Newton's method. According to [39, Section 11.5.3], the number of Newton iterations is upper bounded by $\mathcal{O}\left(\sqrt{m}\right)$. Since the complexity of one Newton iteration is $\mathcal{O}\left(mn^2\right)$, the overall complexity of solving a convex problem is upper bounded by $\mathcal{O}\left(m^{1.5}n^2\right)$. The CCCP algorithm has the largest problem size since it involves optimizing N covariance matrices of size $M \times M$. The number of inequalities in CCCP and DDB are comparable.

In contrast, the proposed DDB algorithm's problem size does not depend on the number of antennas, M, as pointed out in Remark 3. The power allocation problem (43) also does not depend on the number of antennas. Further, its variables are real-valued rather than complex-valued as is the case for the other algorithms. In summary, for large numbers of antennas, the complexity of CCCP and OBPPD-DDB grow by $\mathcal{O}\left(M^{5.5}\right)$ and $\mathcal{O}\left(M^{1.5}\right)$, respectively, and the complexity of OBPPD-RIN does not depend on M. Therefore, the complexity of CCCP is much higher in comparison to OBPPD when the number of antennas gets large.

The average run times for sum rate maximization using CCCP, OBPPD-DDB, and OBPPD-RIN running MATLAB on an Intel(R) Core(TM) i7-12700H, 2.30GHz processor with 16.0 GB of RAM for 2-layer RS with parameters $(M, K, \text{SNR}) = (7, 5, 5\text{dB})$ are, respectively, 31, 6.4, and 2.4 seconds. The order of these run times is consistent with the above per-iteration complexity analysis. To investigate the effect of iteration on complexity, the average number of iterations for the convergence of CCCP, DDB, and OBPPD-RIN's power allocation by approximating the solution to Problem (43) in the same simulation scenario are, respectively, 13.20, 8.48, and 7.53. It is observed that these numbers of iterations are of the same order of magnitude and not very large. This indicates that the above per-iteration complexity analysis captures the essential computational differences among these algorithms. It is also observed that the numbers of iterations correspond to the order of algorithm run times.

TABLE 3. Complexity of sub-problems in precoding design algorithms.

Algorithm	Complexity of each iteration
CCCP [22] (Problem (44))	$\mathcal{O}\left(\left(MN + 2\sum_{i=1}^{N} N_i + 1\right)^{1.5}\left(M^2N + \sum_{i=1}^{N} N_i\right)^2\right)$
DDB (Problem (35))	$\mathcal{O}\left(\left(MN + \sum_{i=1}^{N} N_i\right)^{1.5}\left(\sum_{i=1}^{N} N_i\right)^2\right)$
Power allocation (Problem (43))	$\mathcal{O}\left(\left(N + 2\sum_{i=1}^{N} N_i + 1\right)^{1.5}\left(N + \sum_{i=1}^{N} N_i\right)^2\right)$

C. SIMULATIONS

In the simulations, total transmit power is proportional to the number of users. Denoting average received SNR in dB by γ_{ave}, total transmit power, $P_{\text{tot}} = K\left(10^{\gamma_{\text{ave}}/10}\right)$. Users' channels are modeled as in [20] such that user k's channel vector $\mathbf{h}_k \sim \mathcal{CN}\left(\mathbf{0}, \alpha_k \mathbf{R}_k\right)$, where the channel power gain $\alpha_k, \forall k = 1, \ldots, K$ is Rayleigh distributed with mean $\frac{\pi}{4-\pi}\sigma^2$ and variance, σ^2, a measure of channel power disparity, set to 1. The matrix $\mathbf{R}_k$ is modeled by one-ring scattering [40]. The BS is equipped with a uniform linear array with antenna spacing d such that $u_n = nd - \frac{M+1}{2}d, \forall n = 1, \ldots, M$, is the position of the nth antenna element. Users' angles of arrival, $\theta_k, \forall k \in \mathcal{K}$, are uniformly distributed over $[0, 2\pi)$. The angle spread Δ is $\frac{\pi}{6}$ for all users. Elements of $\mathbf{R}_k$ are [40]:

$$[\mathbf{R}_k]_{n,p} = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e^{-j\frac{2\pi}{\lambda}\cos(x+\theta_k)(u_n-u_p)}\mathrm{d}x, \quad \forall n, \quad p = 1, \ldots, M, \tag{47}$$

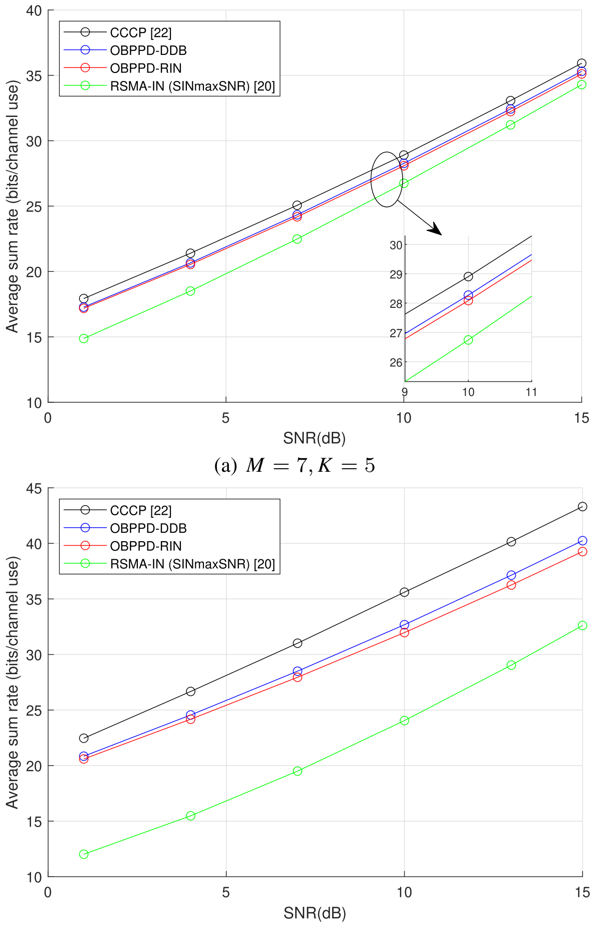
where λ is the carrier wavelength, which is 12.5cm at 2.4GHz, and antenna spacing $d = \frac{\lambda}{2}$.

Figure 2 presents the sum rates of 2-layer RS for two network loads, $(M, K) = \{(7, 5), (7, 7)\}$, evaluated by approximating the solution to the sum rate maximization problem using CCCP [22], OBPPD-DDB, OBPPD-RIN, and RSMA-IN [20]. These network loads correspond to the under-loaded and full-loaded cases, which are compatible with the conditions of Theorem 3, and the comparison with RSMA-IN is due to its good performance and low complexity for under-loaded networks. As mentioned earlier, the results

$$\underset{\substack{\mathbf{r}\succeq\mathbf{0},\mathbf{Q}_i\succeq\mathbf{0},\\ \forall i=1,\ldots,N}}{\text{maximize}} \; f\left(\mathbf{r}\right) \tag{44a}$$

$$\text{subject to} \quad \|\mathbf{r}_i\|_1 - \log_2\left(1 + \frac{\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{Q}_i \mathbf{h}_{\mathrm{l}_i(u)}}{\mathbf{h}_{\mathrm{l}_i(u)}^H\left(\sum_{j=i+1}^{N}\mathbf{Q}_j + \sum_{j=1}^{i-1}\mathbf{Q}_j\delta_{(\mathrm{l}_i(u)\notin\mathcal{I}_j)}\right)\mathbf{h}_{\mathrm{l}_i(u)} + 1}\right) \leq 0,$$
$$\forall u = 1, \ldots, N_i, \quad \forall i = 1, \ldots, N, \tag{44b}$$

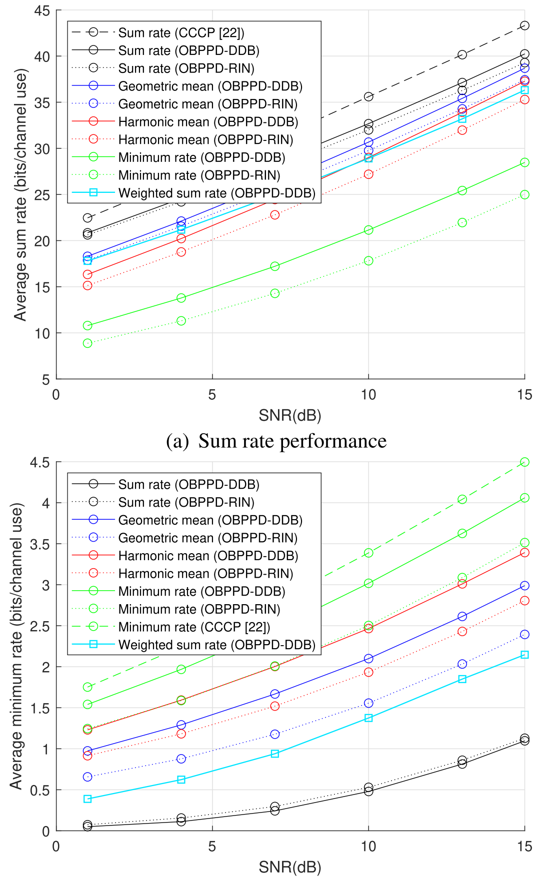
$$\sum_{i=1}^{N}\mathrm{Tr}\left(\mathbf{Q}_i\right) \leq P_{\text{tot}}. \tag{44c}$$



(a) $M = 7, K = 5$

(b) $M = 7, K = 7$

FIGURE 2. **Sum rate maximization problem of 2-layer RS is approximated by RIN, CCCP [22] and RSMA-IN [20] for two network load settings.**



(a) Sum rate performance

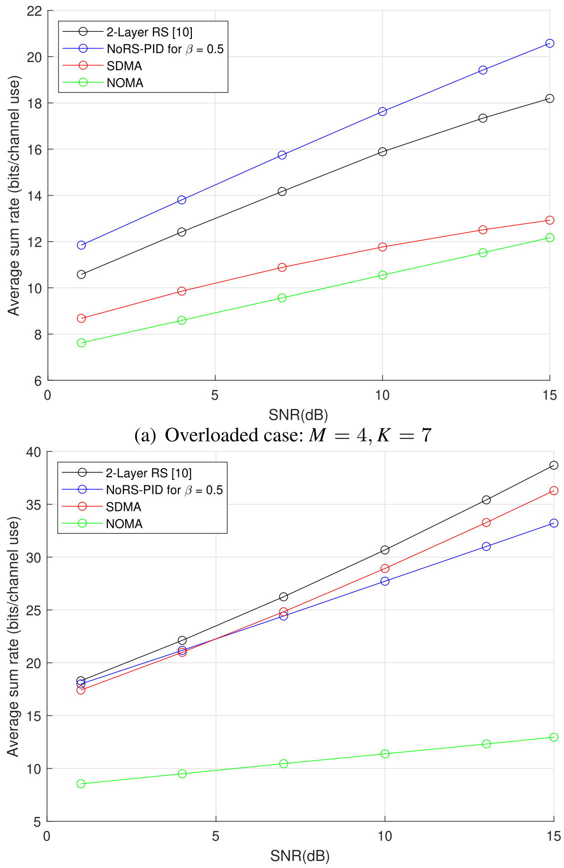
(b) Minimum rate performance

FIGURE 3. **2-Layer RS performance is evaluated by weighted sum rate as well as four objective functions, namely, sum rate, geometric mean, harmonic mean, and max-min.**

of CCCP provide a benchmark for the maximum sum rate, and for both network loads, OBPPD-DDB outperforms OBPPD-RIN due to the proposed Lagrange multiplier selection procedure, Algorithm 2. The sum rate of "RSMA-IN (SIN-MaxSNR)" is lower than that of the other algorithms, whose distance to the optimal sum rate depends on the network load. The lower the network load, the closer the sum rate maximization under interference nulling approaches the optimal sum rate. From Fig. 2, OBPPD-RIN and OBPPD-DDB, by providing a trade-off between interference nulling and maximizing streams' SNR, outperform RSMA-IN sum rate, while providing performance reasonably close to that of CCCP at lower complexity.

Figure 3 shows the sum rate and minimum rate of 2-layer RS with $(M, K) = (7, 7)$, evaluated by approximating the solution to performance maximization Problem $\mathcal{P}1$ using OBPPD-DDB and OBPPD-RIN algorithms for the sum rate, geometric mean, harmonic mean, and max-min objective functions. For comparison purposes, the solution to sum rate maximization and max-min fairness problems approximated by the CCCP algorithm [22], are shown in Figs. 3(a) and (b), respectively. According to Fig. 3, the improvement of OBPPD-DDB over OBPPD-RIN is significant for objective functions other than sum rate, because in sum rate maximization, no stream or user is preferred over others. Therefore, if the channel disparity is small, there is a symmetry among different users/streams, and selecting equal-valued Lagrange multipliers in OBPPD-RIN is near optimal. Sum rate maximization and max-min fairness are two extreme cases, where the former sacrifices fairness and the latter sacrifices aggregate system performance, while geometric mean and harmonic mean provide a trade-off between fairness and sum rate maximization. For example, With OBPPD-RIN precoding, using geometric mean decreases sum rate by 3.8% at 15dB SNR, but increases the minimum rate of users by 173% in comparison to sum rate maximization.

In addition to the four previously mentioned objective functions simulated in Fig. 3, the scenario is also simulated for weighted sum rate using OBPPD-DDB algorithm, where the weighting of user k is $w_k = \frac{\|\mathbf{h}_k\|^{-1}}{\sum_{l=1}^{K}\|\mathbf{h}_l\|^{-1}}, \forall k = 1, \ldots, K$. The rationale for this choice of weights is to enhance the rates of users with poor channel strength, which enhances fairness in comparison to equal weighting, i.e., $w_k = 1, \forall k = 1, \ldots, K$. As presented in Fig. 3(b), the minimum user rate is increased by this weighted sum rate in comparison to the unweighted sum rate, but at a cost of decreasing aggregate performance as shown in Fig. 3(a). Sum rate weighting therefore provides an alternative method to trade off fairness and aggregate performance. It is worth noting that channel strength weighting may be less desirable than employing the harmonic mean. As shown in Fig. 3(a), while aggregate performances of the two methods are similar, the minimum rate provided by harmonic mean is larger as shown in Fig. 3(b).



(a) Overloaded case: $M = 4, K = 7$

(b) Under-loaded case: $M = 7, K = 7$

FIGURE 4. Sum rate 2-layer RS, NoRS-PID, NOMA, and SDMA for overloaded and under-loaded cases is evaluated by geometric mean maximization.

Figure 4 compares the sum rate of 2-layer RS, NoRS-PID, NOMA, and SDMA for network loads $(M, K) = \{(4, 7), (7, 7)\}$. For the overloaded case, $M < K$, SDMA in sum rate maximization problem tends to select M out of K users. Therefore, to ensure that all users have nonzero rate, the geometric mean of users is chosen as the objective function, and OBPPD-DDB is used for precoding design. The parameter β used for NoRS-PID is 0.5. Interference is managed by a combination of *i)* splitting users' rates, *ii)* decoding interference by SIC, and *iii)* using spatial degrees of freedom provided by the antennas. While all schemes exploit spatial degrees of freedom, in the overloaded case, this technique cannot effectively suppress interference. Since SDMA only uses spatial degrees of freedom, its performance degrades in the overloaded case, which agrees with Fig. 4.

From Fig. 4, NoRS-PID is observed to outperform NOMA, because beamforming vectors are in the direction of users decoding that stream, and NOMA tries to decode all interference, which results in wasted signal power to decode interference at all users. Therefore, NOMA cannot effectively use the spatial degrees of freedom provided by the antennas, and its performance is less than that of the other schemes. On the other hand, NoRS-PID provides a trade-off between decoding interference and treating interference as noise. Also from Fig. 4(a), NoRS-PID has a higher sum rate than that of SDMA, since NoRS-PID exploits SIC to manage interference in the overloaded case. Lastly, 2-layer RS applies all 3 interference management approaches and outperforms other schemes with the network load $(M, K) = (7, 7)$. When overloaded, however, as shown in Fig. 4(a), NoRS-PID outperforms 2-layer RS, because its number of SIC layers is restricted to 2, unlike NoRS-PID.

VI. CONCLUSION

An explicit parameterization is obtained for the optimal beamforming directions of a formulated performance maximization problem. These beamforming directions are then applied to proposed OBPPD-DDB and OBPPD-RIN precoding designs that provide a trade-off between interference nulling and maximizing streams' SNRs. The methods outperform the existing RSMA-IN design in sum rate, and offer reasonable performance compared to existing approximations to the optimal solution, but at lower complexity, especially as the number of BS antennas increases.

The OBPPD-DDB and OBPPD-RIN precoders approximate the solution to the performance maximization problem with the four well-known objective functions: i) sum rate, ii) geometric mean rate, iii) harmonic mean rate, and iv) minimum of users' rates, each of which provide a particular trade-off between maximizing aggregate performance and providing fairness. Simulation results indicate that a geometric mean performance objective decreases sum rate by 3.8% at 15 dB SNR while increasing the minimum user's rate by 173% over a sum rate objective.

In the proposed RSMA sub-scheme, NoRS-PID, users do not split their rate, partially decode interference, and subsume SDMA and NOMA stream decoding as special cases. Simulation results show that NoRS-PID effectively exploits the spatial (antenna) domain and interference decoding to outperform SDMA, NOMA and a previous low-complexity version of RSMA, 2-layer RS, in the overloaded case.

APPENDIX A
TOTAL POWER IS STRICTLY INCREASING IN STREAM SINRS

Lemma 1: Assume that $P_1 = \mathcal{P}2(\gamma_1)$ *with the optimal beamforming matrix* $\mathbf{T}_1$ *and* $P_2 = \mathcal{P}2(\gamma_2)$ *with the optimal beamforming matrix* $\mathbf{T}_2$*. Then* $\gamma_1 > \gamma_2$ *results in* $P_1 > P_2$*.*

Proof: The proof is similar to [36, Theorem 3]. Lemma 1 can be proved by contradiction. Assume that $\gamma_1 > \gamma_2$, but $P_1 < P_2$. This assumption is in contradiction with the optimality of $\mathbf{T}_2$ for Problem $\mathcal{P}2\left(\gamma_2\right)$, because a constant $0 < c < 1$ can always be found such that $c\mathbf{T}_1$ satisfies the QoS constraints in Problem $\mathcal{P}2\left(\gamma_2\right)$ with power c^2P_1, which is less than the minimum power required by the beamforming matrix $\mathbf{T}_2$ ($c^2P_1 \leq P_1 \leq P_2$). □

APPENDIX B
FIRST RELATION BETWEEN PROBLEMS $\mathcal{P}1$ AND $\mathcal{P}2$

Theorem 4: Assume that

- $P = \mathcal{P}2\left(\gamma\right)$ *with optimal beamforming matrix* $\mathbf{T}$,
- $f = \mathcal{R}\left(G\left(\mathbf{T}\right)\right)$ *with optimal solution* $\mathbf{r}$, *and*
- $\widetilde{f} = \mathcal{P}1\left(P\right)$ *with optimal beamforming matrix* $\widetilde{\mathbf{T}}$ *and optimal split rate vector* $\widetilde{\mathbf{r}}$, *where* $\widetilde{\mathbf{r}}$ *is the solution to* $\mathcal{R}\left(\widetilde{\gamma}\right)$ *such that* $\widetilde{\gamma} = G\left(\widetilde{\mathbf{T}}\right)$.

Then $\widetilde{f} = f$, *and one solution for* $\mathcal{P}1\left(P\right)$ *is* $\widetilde{\mathbf{T}} = \mathbf{T}$ *and* $\widetilde{\mathbf{r}} = \mathbf{r}$.

Proof: By contradiction assume that $\widetilde{f} \neq f$. Therefore, there are two cases: *(i)* $\widetilde{f} < f$ and *(ii)* $\widetilde{f} > f$. According to monotonicity results of Lemma 2 below, cases (i) and (ii) result in $\widetilde{\gamma} < \gamma$ and $\widetilde{\gamma} > \gamma$, respectively.

Case (i) is in contradiction with the optimality of $\widetilde{\mathbf{T}}$ for Problem $\mathcal{P}1\left(P\right)$, because the beamforming matrix $\mathbf{T}$ can achieve a higher objective value f, while it does not violate the power constraint in $\mathcal{P}1\left(P\right)$.

Case (ii) is in contradiction with the optimality of $\mathbf{T}$ for Problem $\mathcal{P}2\left(\gamma\right)$, because $\widetilde{f} > f$ results in $\widetilde{\gamma} > \gamma$, and we can always find $0 < c < 1$ such that $c\widetilde{\mathbf{T}}$ does not violate the QoS SINR in Problem $\mathcal{P}2\left(\gamma\right)$, while its required power is c^2P, which is less than the minimum power achieved by the beamforming matrix $\mathbf{T}$.

The fact that neither cases (i) and (ii) can occur establishes that $\widetilde{f} = f$. The rest of Theorem 4, which states that one solution for Problem $\mathcal{P}1\left(P\right)$ is $\widetilde{\mathbf{T}} = \mathbf{T}$ and $\widetilde{\mathbf{r}} = \mathbf{r}$ can be concluded from two facts. First, the beamforming matrix $\mathbf{T}$ is the solution to $P = \mathcal{P}2\left(\gamma\right)$ and therefore does not violate the power constraint of Problem $\mathcal{P}1\left(P\right)$. Second, as the QoS constraints of Problem $\mathcal{P}2\left(\gamma\right)$ are active at its optimal point, the matrix $\mathbf{T}$ achieves the streams' SINR vector γ, and $f = \mathcal{R}\left(\gamma\right)$ with the optimal solution $\mathbf{r}$ achieving the optimal value of Problem $\mathcal{P}1\left(P\right)$. □

Lemma 2: Assume that $f_1 = \mathcal{R}\left(\gamma_1\right)$ *with the optimal solution* $\mathbf{r}_1$ *and* $f_2 = \mathcal{R}\left(\gamma_2\right)$ *with the optimal solution* $\mathbf{r}_2$. *Then* $f_1 > f_2$ *results in* $\gamma_1 > \gamma_2$.

Proof: (Lemma 2). This follows from the fact that $f\left(\mathbf{r}\right)$ is an strictly increasing of $\mathbf{r}$. According to this fact, at the optimal point of Problem $\mathcal{R}$, the constraints in (10b) must be satisfied with equality. Therefore, based on the N equations of (10b), $[\mathbf{Br}]_i = \log_2\left(1+\gamma_i\right), \forall i = 1, \ldots, N$, where $\mathbf{B}$ is of size $N \times \sum_{i=1}^{N} N_i$ with elements

$$[\mathbf{B}]_{i,j} = \begin{cases} 1, & j = \sum_{l=1}^{i-1} N_l + 1, \ldots, \sum_{l=1}^{i} N_l, \\ 0, & O.W., \end{cases} \quad \forall i = 1, \ldots, N. \tag{48}$$

As $f\left(\mathbf{r}\right)$ is an strictly increasing of $\mathbf{r}$, $f_1 > f_2$ results in $\mathbf{r}_1 > \mathbf{r}_2$. Since all elements of $\mathbf{B}$ are non-negative, this also results in $\mathbf{Br}_1 > \mathbf{Br}_2$. Note that since certain elements of $\mathbf{B}$ are greater than 0 and $\mathbf{r}_1 > \mathbf{r}_2$, the inequality $\mathbf{Br}_1 > \mathbf{Br}_2$ is strict. Finally, as logarithm is a strictly increasing function, it is concluded that $\gamma_1 > \gamma_2$. □

APPENDIX C
SECOND RELATION BETWEEN PROBLEMS $\mathcal{P}1$ AND $\mathcal{P}2$

Theorem 5: Assume that

- $f = \mathcal{P}1\left(P\right)$ *with optimal beamforming matrix* $\mathbf{T}$ *and optimal split rate vector* $\mathbf{r}$, *where* $\mathbf{r}$ *is the solution to* $\mathcal{R}\left(\gamma\right)$ *such that* $\gamma = G\left(\mathbf{T}\right)$ *and*
- $\widetilde{P} = \mathcal{P}2\left(\gamma\right)$ *with optimal beamforming matrix* $\widetilde{\mathbf{T}}$.

Then $\widetilde{P} = P$, *and one solution for* $\mathcal{P}2\left(\gamma\right)$ *is* $\widetilde{\mathbf{T}} = \mathbf{T}$.

Proof: Theorem 5 can be proved by contradiction. Assume that $\widetilde{P} \neq P$. Therefore, one of the two cases *(i)* $\widetilde{P} > P$ and *(ii)* $\widetilde{P} < P$ may occur.

Case (i) is in contradiction with the optimality of $\widetilde{\mathbf{T}}$ for Problem $\mathcal{P}2\left(\gamma\right)$, because the beamforming matrix $\mathbf{T}$ can achieve SINR vector γ, as a result, the QoS constraints in Problem $\mathcal{P}2\left(\gamma\right)$ are satisfied, while the total power required for the beamforming matrix $\mathbf{T}$ is P, which is less than $\widetilde{P}$.

It can be shown that Case (ii) is also in contradiction with the optimality of $\mathbf{T}$ for Problem $\mathcal{P}1\left(P\right)$. All QoS constraints of Problem $\mathcal{P}2\left(\gamma\right)$ at the optimal point are active. Therefore, the beamforming matrix $\widetilde{\mathbf{T}}$ can achieve SINR vector γ, while its required total power is $\widetilde{P}$, which is less than the total power limit in Problem $\mathcal{P}1\left(P\right)$. Multiplying $\widetilde{\mathbf{T}}$ by $c = \sqrt{P/\widetilde{P}} > 1$, we obtain the beamforming matrix $c\widetilde{\mathbf{T}}$ that achieves SINR vector $\overline{\gamma}$ that is greater than γ with the total power $c^2\widetilde{P} = P$. According to Lemma 3 below, since $\overline{\gamma} > \gamma$, the objective function of Problem $\mathcal{P}1\left(P\right)$ is greater with the beamforming matrix $c\widetilde{T}$ without violating the power constraint, which is in contradiction with the optimality of $\mathbf{T}$ for Problem $\mathcal{P}1\left(P\right)$.

As neither cases (i) and (ii) can occur, it is established that $\widetilde{P} = P$. The rest of Theorem 5, which states that one solution for Problem $\mathcal{P}2\left(\gamma\right)$ is $\widetilde{\mathbf{T}} = \mathbf{T}$ can be concluded from two facts. First, the matrix $\mathbf{T}$ achieves the streams' SINR vector γ. Therefore, it does not violate the QoS constraints of Problem $\mathcal{P}2\left(\gamma\right)$. Second, as $\mathbf{T}$ is the optimal solution to Problem $\mathcal{P}1\left(P\right)$ and the power constraint of Problem $\mathcal{P}1\left(P\right)$ is active at its optimal point, the total power of beamforming matrix $\mathbf{T}$ is P, which is the same as the optimal value of Problem $\mathcal{P}2\left(\gamma\right)$. □

Lemma 3: Assume that $f_1 = \mathcal{R}\left(\gamma_1\right)$ *with the optimal solution* $\mathbf{r}_1$ *and* $f_2 = \mathcal{R}\left(\gamma_2\right)$ *with the optimal solution* $\mathbf{r}_2$. *Then* $\gamma_1 > \gamma_2$ *results in* $f_1 \geq f_2$.

Proof: (Lemma 3) By contradiction assume that $f_1 < f_2$. This is in contradiction with the optimality of $\mathbf{r}_1$ for Problem $\mathcal{R}\left(\gamma_1\right)$, because $\mathbf{r}_2$ satisfies the QoS SINRs of Problem $\mathcal{R}\left(\gamma_1\right)$, while its objective value f_2 is greater than the objective value obtained by $\mathbf{r}_1$. □

APPENDIX D
PROOF OF THEOREM 1

Based on [41, Theorem 12.1], if at a local optimum of Problem (12), linear independence constraint qualification (LICQ) holds, KKT conditions hold at that local optimum. Therefore, proving that LICQ holds at any point of Problem (12) results in the necessity of KKT conditions at any local optimum. According to [41, Definition 12.4], LICQ holds at a given point of Problem (12) if the set of active constraint gradients is linearly independent. In the following, we will show that LICQ is satisfied at any point of Problem (12) for a full column rank channel matrix.

Let us represent the constraints in (12b) by $C_{i,u}, \forall u = 1, \dots, N_i, \forall i = 1, \dots, N$. Differentiating,

$$\frac{\partial C_{i,u}}{\partial \mathbf{q}_t^*} = \begin{cases} \mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_t \delta_{(\mathrm{l}_i(u)\notin \mathcal{I}_t)}, & t = 1, \dots, i-1, \\ -\frac{1}{\overline{\gamma}_i}\mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_t, & t = i, \\ \mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_t, & t = i+1, \dots, N, \end{cases} \quad \forall u = 1, \dots, N_i, \forall i = 1, \dots, N. \tag{49}$$

Therefore, the gradient of the constraint $C_{i,u}$ is as follows:

$$\nabla C_{i,u} = \mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \Big[\mathbf{q}_1 \delta_{(\mathrm{l}_i(u)\notin \mathcal{I}_1)}, \dots, \mathbf{q}_{i-1}\delta_{(\mathrm{l}_i(u)\notin \mathcal{I}_{i-1})}, -\frac{1}{\overline{\gamma}_i}\mathbf{q}_i, \mathbf{q}_{i+1}, \dots, \mathbf{q}_N\Big], \forall u = 1, \dots, N_i, \forall i = 1, \dots, N. \tag{50}$$

Without loss of generality, we can assume that the optimization variable of (12) is $\widetilde{\mathbf{q}} = \left[\mathbf{q}_1^T, \dots, \mathbf{q}_N^T\right]^T$. Therefore, the gradients of constraints, $\nabla_{\widetilde{\mathbf{q}}} C_{i,u}, \forall u = 1, \dots, N_i, \forall i = 1, \dots, N$, become vectors instead of matrices:

$$\nabla_{\widetilde{\mathbf{q}}} C_{i,u} = \begin{bmatrix} \mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_1 \delta_{(\mathrm{l}_i(u)\notin \mathcal{I}_1)} \\ \vdots \\ \mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_{i-1} \delta_{(\mathrm{l}_i(u)\notin \mathcal{I}_{i-1})} \\ -\frac{1}{\overline{\gamma}_i}\mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_i \\ \mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_{i+1} \\ \vdots \\ \mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_N \end{bmatrix}_{MN\times 1}, \tag{51}$$

To prove LICQ, it is required to show that the linear combination of active constraints gradients at the optimal point cannot be zero, unless all multipliers are zero. Consider the worst cast that all constraints are active. Suppose there exist a linear combination of constraint gradients that equal zero:

$$\textstyle\sum_{i=1}^N \sum_{u=1}^{N_i} a_{i,u} \nabla_{\widetilde{\mathbf{q}}} C_{i,u} = \mathbf{0}_{MN\times 1}. \tag{52}$$

The jth M elements of (52) can be expressed as a linear combination of users' channel vectors:

$$\begin{aligned} &\textstyle\sum_{i=1}^N \sum_{u=1}^{N_i} a_{i,u} \left[\nabla_{\widetilde{\mathbf{q}}} C_{i,u}\right]_{(j-1)M+1:jM} \\ &= \textstyle\sum_{i=1}^{j-1} \sum_{u=1}^{N_i} a_{i,u} \mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_j - \sum_{u=1}^{N_j} \frac{a_{j,u}}{\overline{\gamma}_j} \mathbf{h}_{\mathrm{l}_j(u)}\mathbf{h}_{\mathrm{l}_j(u)}^H \mathbf{q}_j \\ &\quad + \textstyle\sum_{i=j+1}^N \sum_{u=1}^{N_i} a_{i,u} \mathbf{h}_{\mathrm{l}_i(u)}\mathbf{h}_{\mathrm{l}_i(u)}^H \mathbf{q}_j \delta_{(\mathrm{l}_i(u)\notin \mathcal{I}_j)}. \end{aligned} \tag{53}$$

By multiplying the summand with indices of $i, \forall i = 1, \dots, N$ and $u = 1, \dots, N_i$ by $\sum_{k=1}^K \delta_{(k=\mathrm{l}_i(u))} = 1$, the linear combination becomes

$$\begin{aligned} &\textstyle\sum_{i=1}^N \sum_{u=1}^{N_i} a_{i,u} \left[\nabla_{\widetilde{\mathbf{q}}} C_{i,u}\right]_{(j-1)M+1:jM} = \sum_{k=1}^K \left[\mathbf{b}_j\right]_k \mathbf{h}_k \\ &= \mathbf{H}\mathbf{b}_j = \mathbf{0}_{M\times 1}, \quad \forall j = 1, \dots, N, \end{aligned} \tag{54}$$

where $\left[\mathbf{b}_j\right]_k, \forall k = 1, \dots, K$ is the kth element of vector $\mathbf{b}_j, \forall j = 1, \dots, N$, which is defined as

$$\begin{aligned} \left[\mathbf{b}_j\right]_k &= \mathbf{h}_k^H \mathbf{q}_j \Bigg(\sum_{i=1}^{j-1}\sum_{u=1}^{N_i} a_{i,u}\delta_{(k=\mathrm{l}_i(u))} - \frac{1}{\overline{\gamma}_j}\sum_{u=1}^{N_j} a_{j,u}\delta_{(k=\mathrm{l}_j(u))} \\ &\quad + \delta_{(k\notin \mathcal{I}_j)} \sum_{i=j+1}^{N}\sum_{u=1}^{N_i} a_{i,u}\delta_{(k=\mathrm{l}_i(u))}\Bigg) = \mathbf{h}_k^H \mathbf{q}_j \Bigg(\sum_{i=1}^{j-1} a_{i,\bar{\mathrm{l}}_i(k)} \\ &\quad - \frac{1}{\overline{\gamma}_j} a_{j,\bar{\mathrm{l}}_j(k)} + \delta_{(k\notin \mathcal{I}_j)} \sum_{i=j+1}^{N} a_{i,\bar{\mathrm{l}}_i(k)}\Bigg). \end{aligned} \tag{55}$$

Considering that $\mathbf{H}$ is of full column rank, to satisfy (53), we must have

$$\begin{aligned} &\mathbf{b}_j = \mathbf{0}_{K\times 1}, \forall j = 1, \dots, N, \\ &\equiv \begin{cases} \displaystyle\sum_{i=1}^{j-1} a_{i,\bar{\mathrm{l}}_i(k)} - \frac{1}{\overline{\gamma}_j} a_{j,\bar{\mathrm{l}}_j(k)} = 0, & \forall k \in \mathcal{I}_j, \\ \displaystyle\mathbf{h}_k^H \mathbf{q}_j \sum_{\substack{i=1,\\ i\neq j}}^{N} a_{i,\bar{\mathrm{l}}_i(k)} = 0, & \forall k \in \{1, \dots, K\} \backslash \mathcal{I}_j, \end{cases} \\ &\forall j = 1, \dots, N, \end{aligned} \tag{56}$$

where the equivalence in (56) is from the fact that since the jth stream is intended for the users in $\mathcal{I}_j$, the product $\mathbf{h}_k^H \mathbf{q}_j, \forall k \in \mathcal{I}_j$ cannot be zero. Therefore, from (56), the following equations can be obtained for the multipliers of the linear combination in (52):

$$\frac{1}{\overline{\gamma}_j} a_{j,\bar{\mathrm{l}}_j(k)} = \textstyle\sum_{i=1}^{j-1} a_{i,\bar{\mathrm{l}}_i(k)}, \quad \forall k \in \mathcal{I}_j, \forall j = 1, \dots, N. \tag{57}$$

According to (57), $a_{1,\bar{\mathrm{l}}_1(k)} = 0, \forall k \in \mathcal{I}_1$. It is straightforward to show that using induction and exploiting the latter point as its base case, we obtain $a_{j,\bar{\mathrm{l}}_j(k)} = 0, \forall k \in \mathcal{I}_j, \forall j = 1, \dots, N$, which means that the only linear combination of gradient vectors that equal the zero vector is zero. Hence, for a full column rank channel matrix, LICQ is satisfied for (12).

APPENDIX E
PROOF OF THEOREM 2

According to [39], the KKT conditions include primal problem constraints, (13); dual problem constraints, (14)-(15); stationary condition, (16); and complementary slackness, (17), where primal constraints and complementary slackness are expressed according to the definition in Theorem 2. To obtain dual problem constraints and stationary condition, dual problem of (12) is obtained in the following. The Lagrangian of Problem (12) can be expressed as

$$\mathcal{L}_{\text{RSMA}}\left(\{\mathbf{q}_i\}_{i=1}^N, \{\lambda_{i,u}, \forall u=1,\ldots,N_i\}_{i=1}^N\right) = \sum_{i=1}^N \mathbf{q}_i^H\mathbf{q}_i$$
$$+ \sum_{i=1}^N \sum_{u=1}^{N_i} \lambda_{i,u}\Bigg(1 + \sum_{j=i+1}^N \mathbf{q}_j^H \mathbf{h}_{\mathsf{l}_i(u)}\mathbf{h}_{\mathsf{l}_i(u)}^H \mathbf{q}_j$$
$$+ \sum_{j=1}^{i-1} \mathbf{q}_j^H \mathbf{h}_{\mathsf{l}_i(u)}\mathbf{h}_{\mathsf{l}_i(u)}^H \mathbf{q}_j \delta_{(\mathsf{l}_i(u)\notin\mathcal{I}_j)} - \frac{1}{\overline{\gamma}_i}\mathbf{q}_i^H \mathbf{h}_{\mathsf{l}_i(u)}\mathbf{h}_{\mathsf{l}_i(u)}^H \mathbf{q}_i\Bigg)$$
$$= \sum_{i=1}^N \mathbf{q}_i^H\left(\mathbf{B}_i - \frac{1}{\overline{\gamma}_i}\mathbf{A}_i\right)\mathbf{q}_i + \sum_{i=1}^N \sum_{u=1}^{N_i} \lambda_{i,u}, \tag{58}$$

where the notation $\{.\}_{i=1}^N$ is used to represent $\forall i = 1,\ldots,N$. The matrices $\mathbf{A}_i$ and $\mathbf{B}_i$, $\forall i = 1,\ldots,N$, are defined in (18) and (19), respectively. The stationary conditions in (16) are obtained by taking derivative of the Lagrangian function, i.e., $\partial\mathcal{L}_{\text{RSMA}}/\partial\mathbf{q}_i^* = 0, \forall i = 1,\ldots,N$.

The Lagrange dual function is the infimum of the Lagrangian function, which can be expressed as

$$g\left(\{\lambda_{i,u}, \forall u=1,\ldots,N_i\}_{i=1}^N\right)$$
$$= \inf_{\mathbf{q}_i,\forall i=1,\ldots,N} \mathcal{L}_{\text{RSMA}}\left(\{\mathbf{q}_i\}_{i=1}^N, \{\lambda_{i,u}, \forall u=1,\ldots,N_i\}_{i=1}^N\right)$$
$$= \begin{cases} \sum_{i=1}^N \sum_{u=1}^{N_i} \lambda_{i,u}, & \left\{\mathbf{B}_i - \dfrac{1}{\overline{\gamma}_i}\mathbf{A}_i \succeq 0\right\}_{i=1}^N, \\ -\infty, & \text{O.W.} \end{cases} \tag{59}$$

According to Lemma 4, $\mathbf{B}_i - \frac{1}{\overline{\gamma}_i}\mathbf{A}_i \succeq 0$ is equivalent to $\max_{\mathbf{q}_i} \frac{\mathbf{q}_i^H\mathbf{A}_i\mathbf{q}_i}{\mathbf{q}_i^H\mathbf{B}_i\mathbf{q}_i} \le \overline{\gamma}_i$.

Lemma 4: For positive semidefinite matrices $\mathbf{A} \in \mathbb{C}^{n\times n}$ *and* $\mathbf{B} \in \mathbb{C}^{n\times n}$, $\mathbf{A} - \mathbf{B} \succeq 0$ *is equivalent to* $\max_{\mathbf{x}} \frac{\mathbf{x}^H\mathbf{B}\mathbf{x}}{\mathbf{x}^H\mathbf{A}\mathbf{x}} \le 1$.

Proof: From the definition of positive semidefinite matrices,

$$\mathbf{A} - \mathbf{B} \succeq 0 \leftrightarrow \mathbf{x}^H(\mathbf{A} - \mathbf{B})\mathbf{x} \ge 0, \forall \mathbf{x} \in \mathbb{C}^{n\times 1}. \tag{60}$$

The right side of (60) is equivalent to the following inequality:

$$\frac{\mathbf{x}^H\mathbf{B}\mathbf{x}}{\mathbf{x}^H\mathbf{A}\mathbf{x}} \le 1, \forall \mathbf{x} \in \mathbb{C}^{n\times 1} \equiv \max_{\mathbf{x}} \frac{\mathbf{x}^H\mathbf{B}\mathbf{x}}{\mathbf{x}^H\mathbf{A}\mathbf{x}} \le 1. \tag{61}$$

□

Based on Lemma 4 and (59), the dual problem of Problem (12) is formulated as

$$\underset{\substack{\lambda_{i,u}\ge 0,\\ \forall u=1,\ldots,N_i,\\ \forall i=1,\ldots,N}}{\text{maximize}} \sum_{i=1}^N \sum_{u=1}^{N_i} \lambda_{i,u} \tag{62a}$$

$$\text{subject to} \max_{\mathbf{q}_i} \frac{\mathbf{q}_i^H\mathbf{A}_i\mathbf{q}_i}{\mathbf{q}_i^H\mathbf{B}_i\mathbf{q}_i} \le \overline{\gamma}_i, \quad \forall i = 1,\ldots,N, \tag{62b}$$

which results in dual problem constraints in (14)-(15).

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